Basic Data Structures: Arrays and Linked Lists

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Data Structures
http://bit.ly/algospecialization

Outline

```
long arr[] = new long[5];
```

```
long arr[5];
```

```
arr = [None] * 5
```

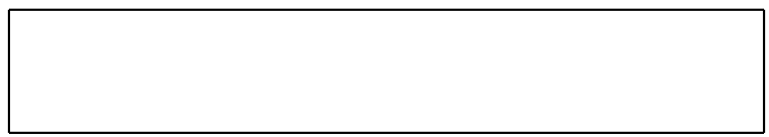
1	5	17	3	25

1	5	17	3	25
8	2	36	5	3

Definition

Array:

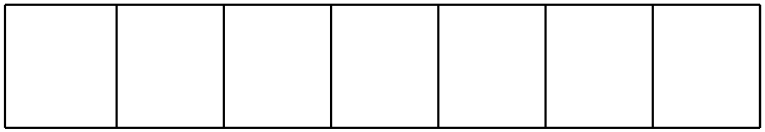
Contiguous area of memory



Definition

Array:

Contiguous area of memory consisting of equal-size elements



Definition

Array:

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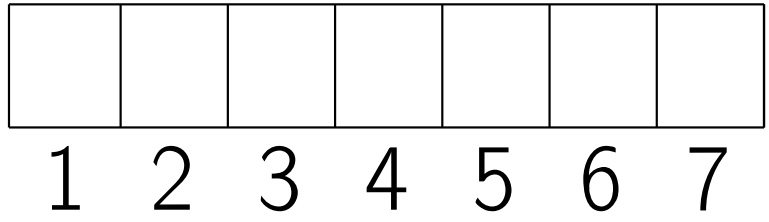
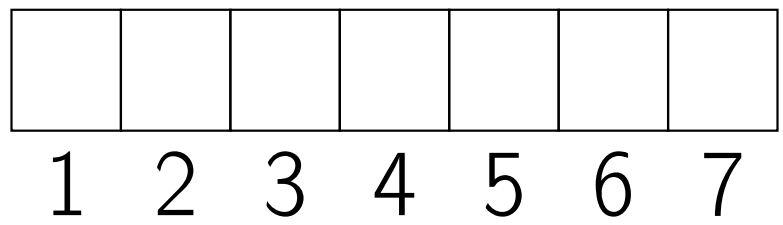
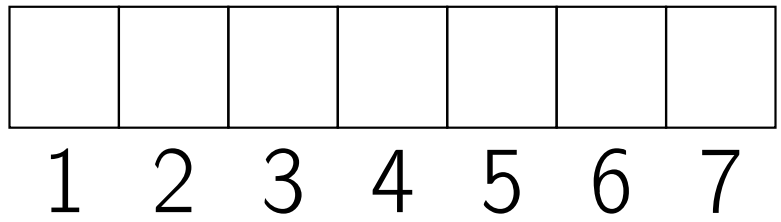


What's Special About Arrays?



What's Special About Arrays?

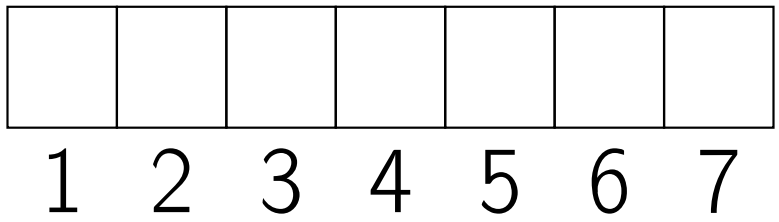
Constant-time access



What's Special About Arrays?

Constant-time access

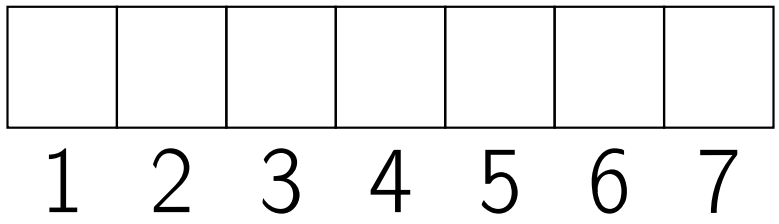
array_addr



What's Special About Arrays?

Constant-time access

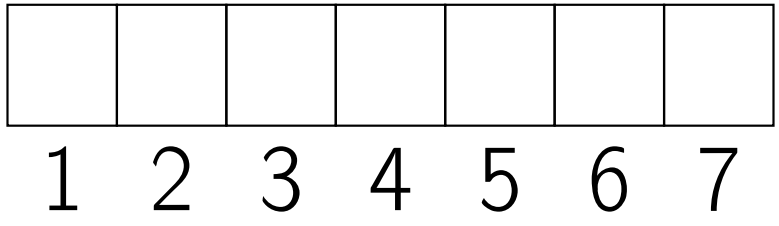
$\text{array\_addr} + \text{elem\_size} \times (\qquad\qquad\qquad\qquad\qquad\qquad)$



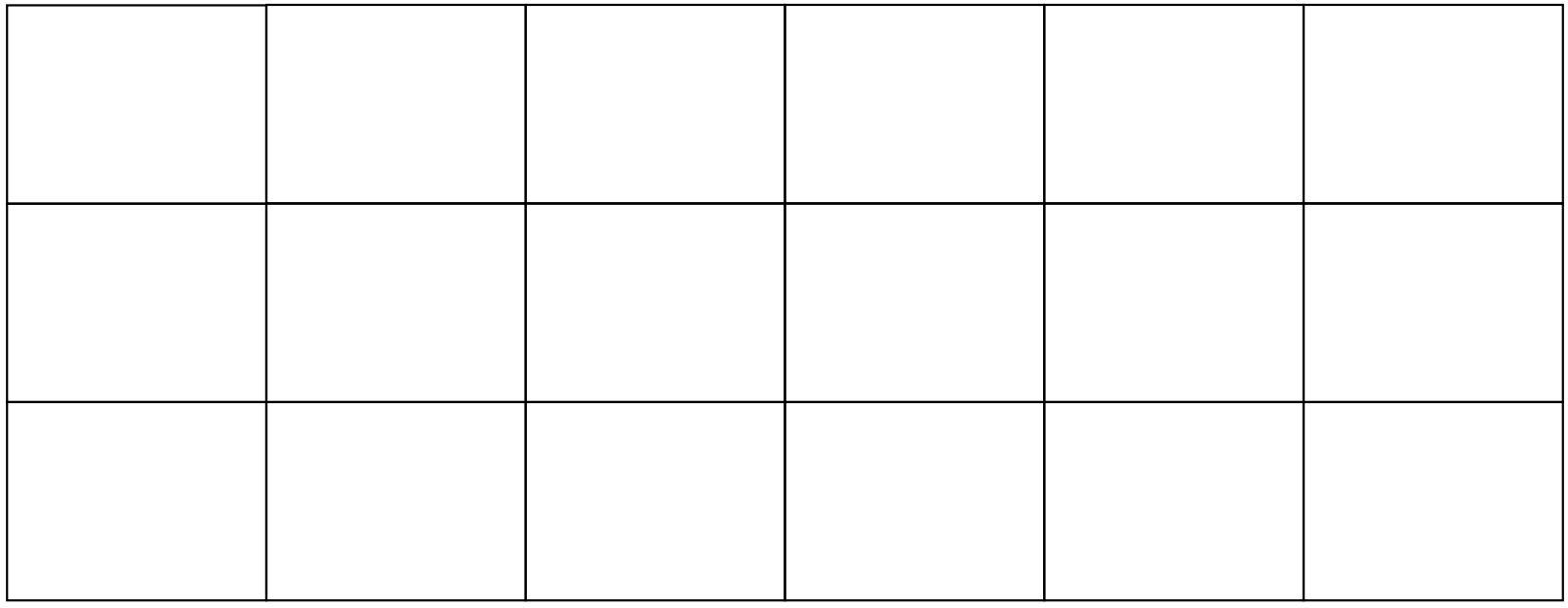
What's Special About Arrays?

Constant-time access

$$\text{array\_addr} + \text{elem\_size} \times (i - \text{first\_index})$$



Multi-Dimensional Arrays



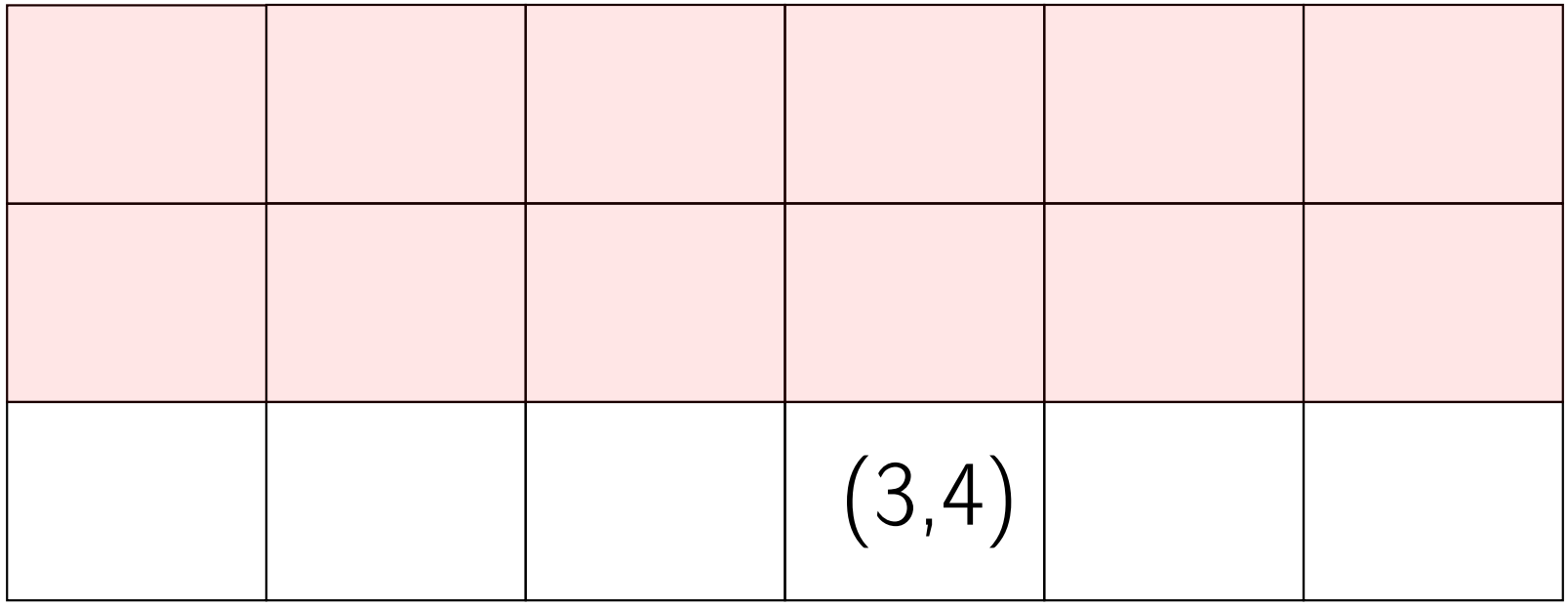
Multi-Dimensional Arrays

(1, 1)					

Multi-Dimensional Arrays

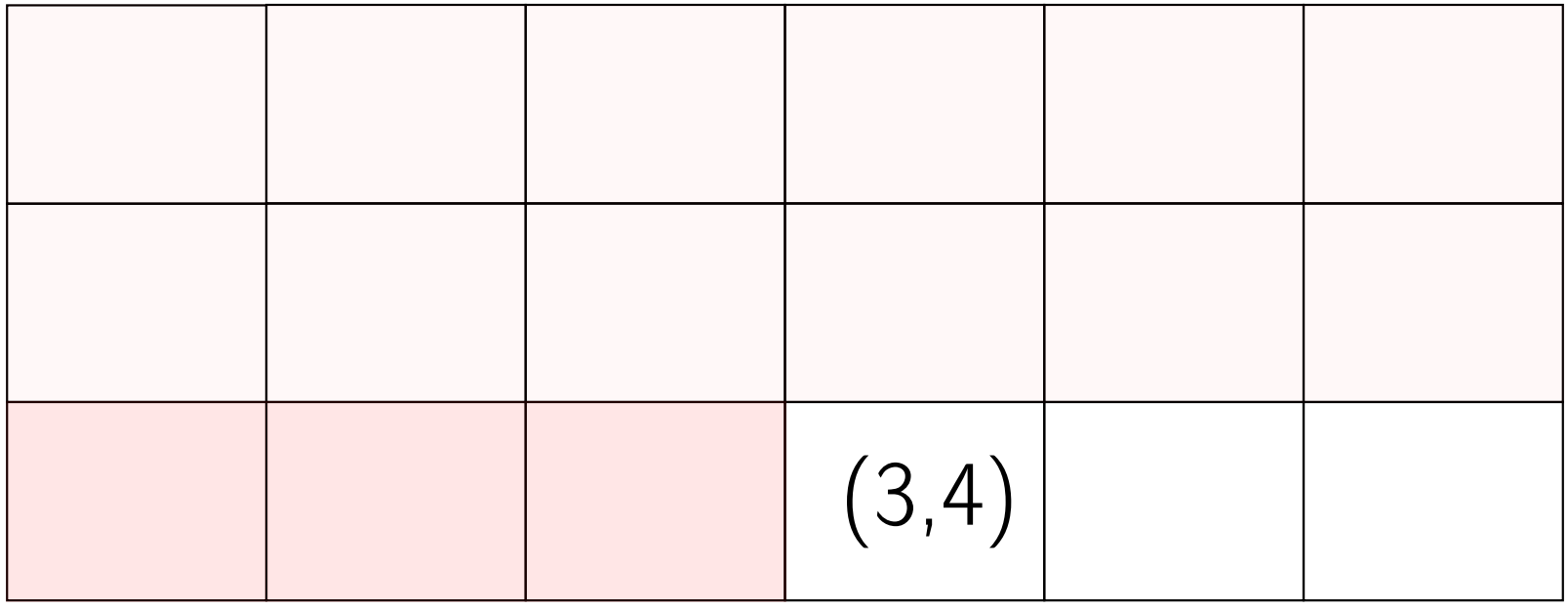
			(3,4)		

Multi-Dimensional Arrays



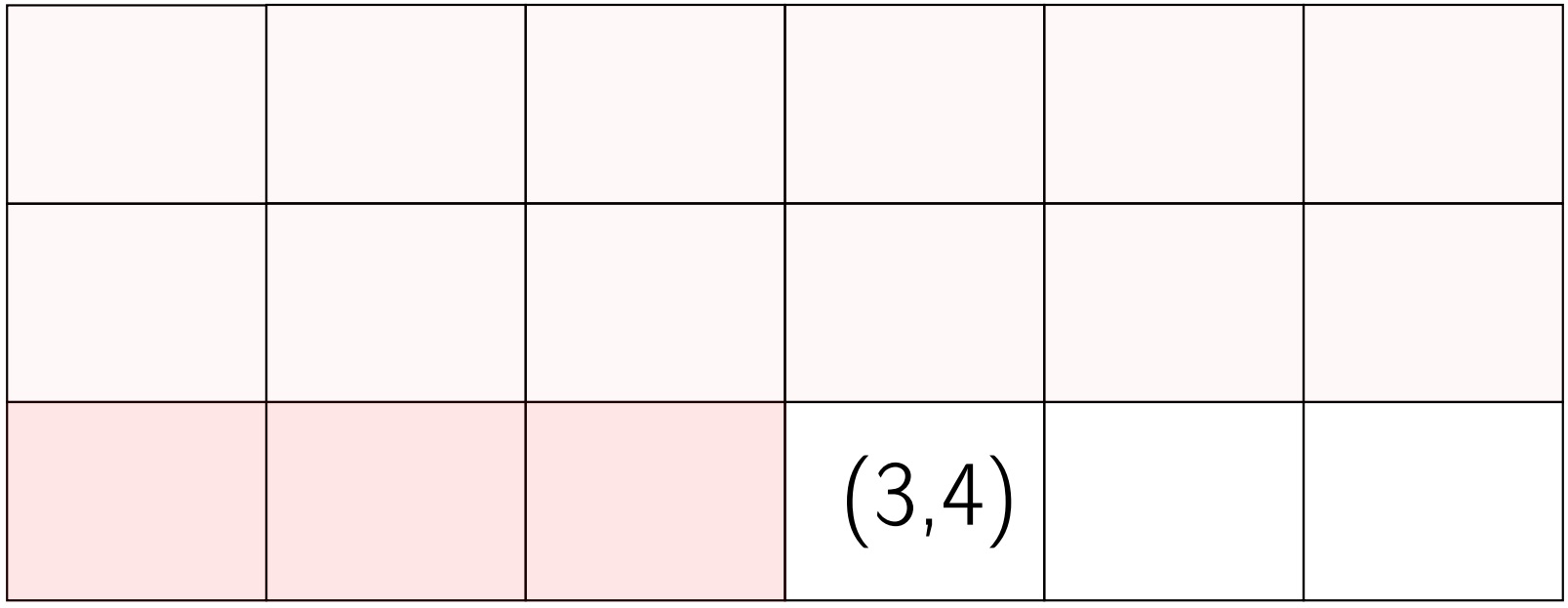
$(3-1) \times 6$

Multi-Dimensional Arrays



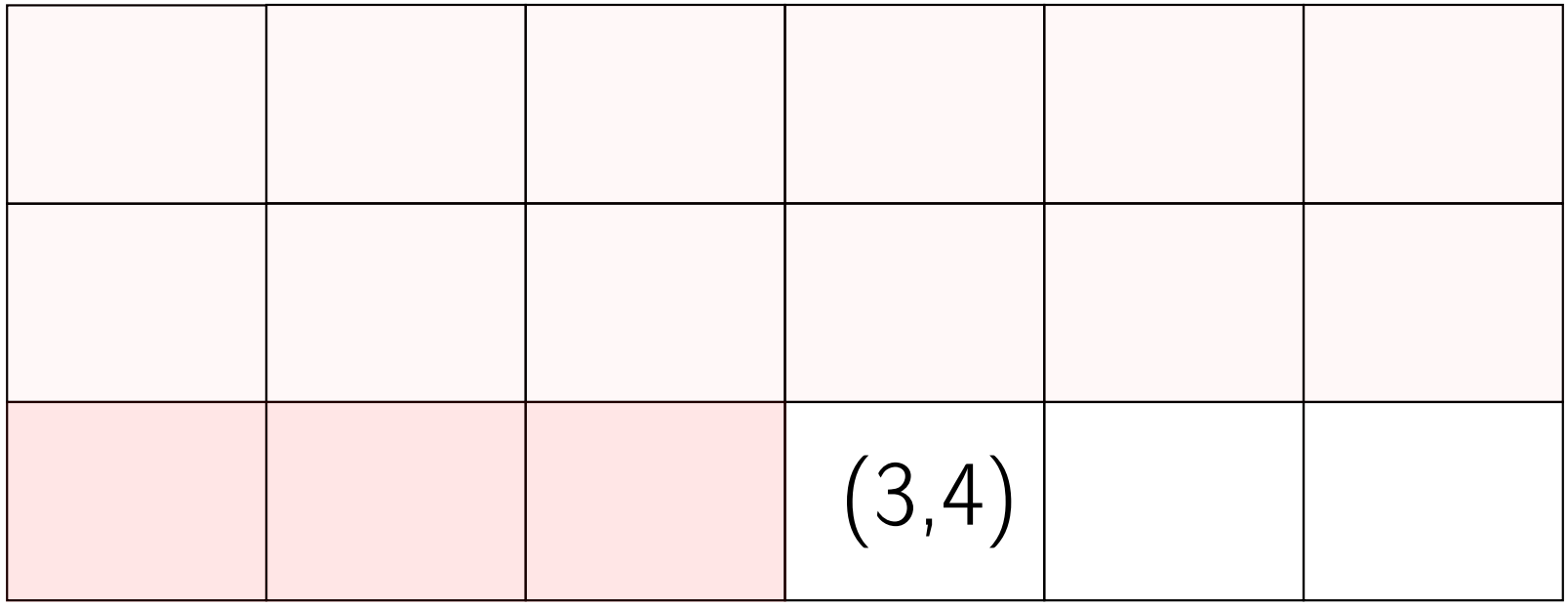
$$(3-1) \times 6 + (4-1)$$

Multi-Dimensional Arrays

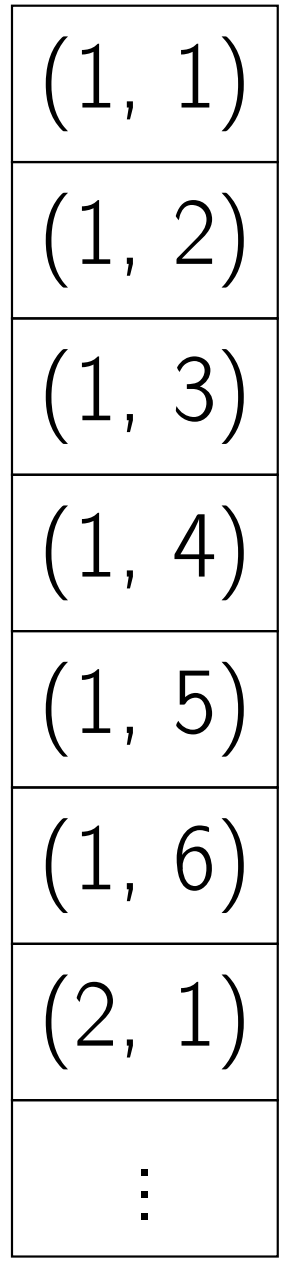


$$\text{elem\_size} \times ((3-1) \times 6 + (4-1))$$

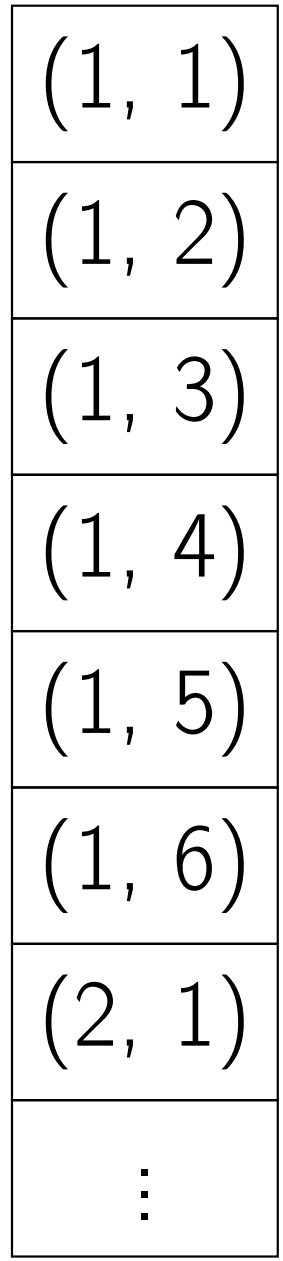
Multi-Dimensional Arrays

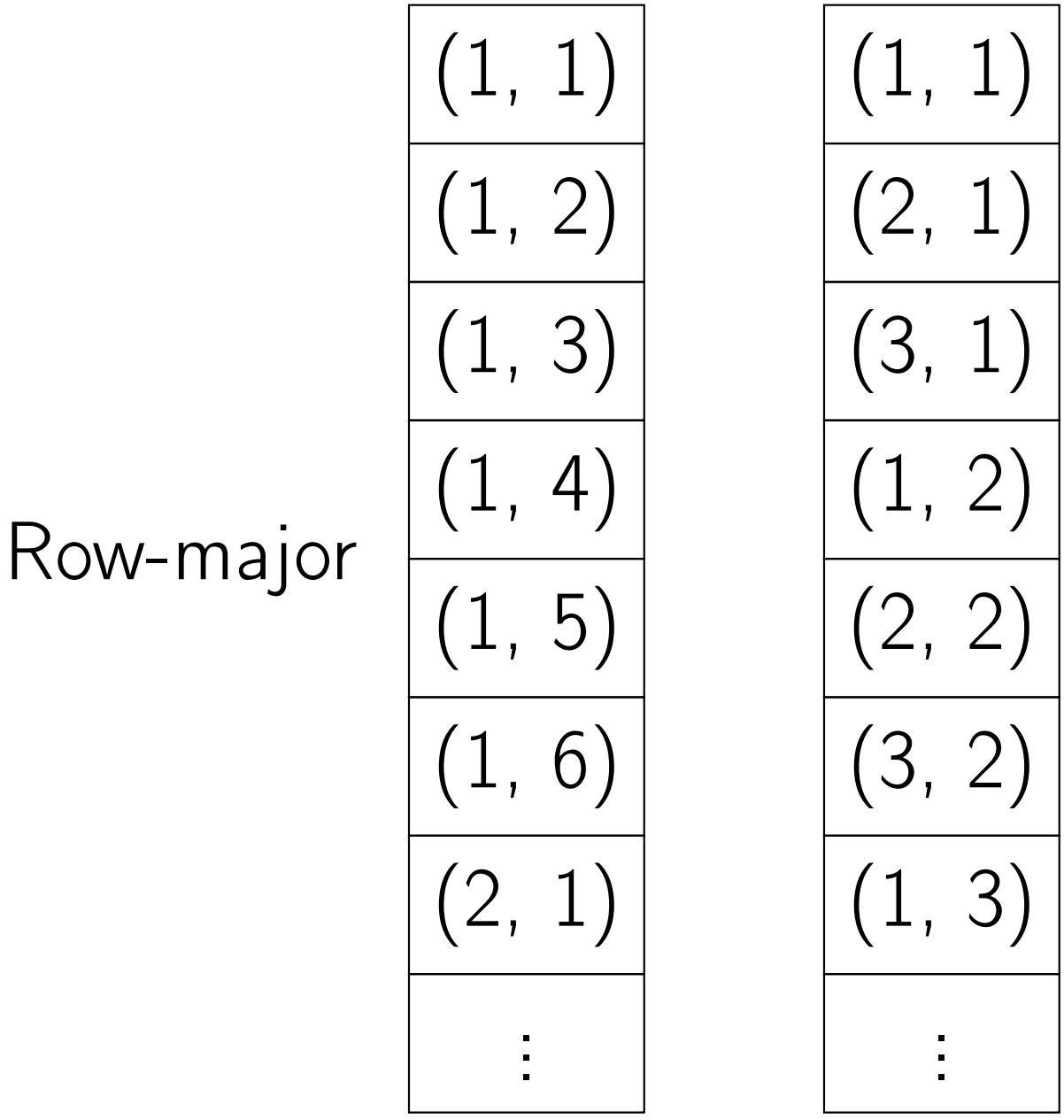


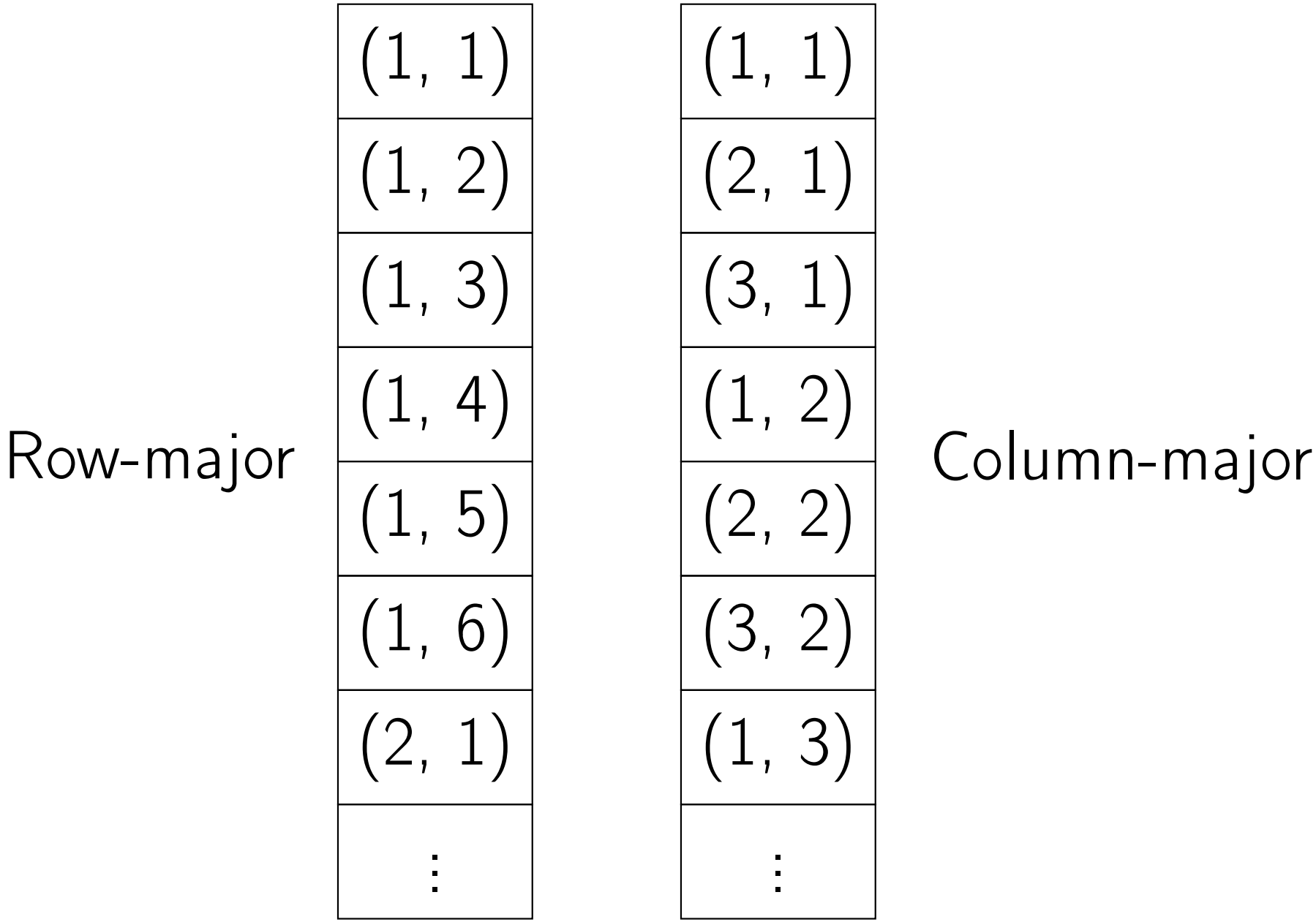
$$\text{array\_addr} +$$
$$\text{elem\_size} \times ((3-1) \times 6 + (4-1))$$


(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
⋮

Row-major




Row-major
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
⋮
(1, 1)
(2, 1)
(3, 1)
(1, 2)
(2, 2)
(3, 2)
(1, 3)
⋮


Row-major
(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
⋮
(1, 1)
(2, 1)
(3, 1)
(1, 2)
(2, 2)
(3, 2)
(1, 3)
⋮
Column-major

Times for Common Operations

	Add	Remove
Beginning		
End		
Middle		

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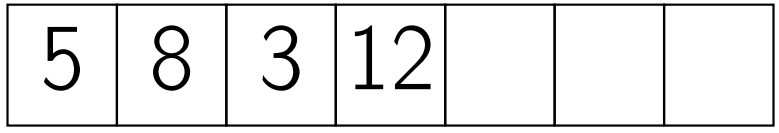
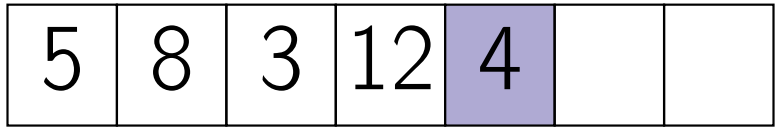
Beginning		
End		
Middle		



Times for Common Operations

	Add	Remove
Beginning		
End	$O(1)$	
Middle		



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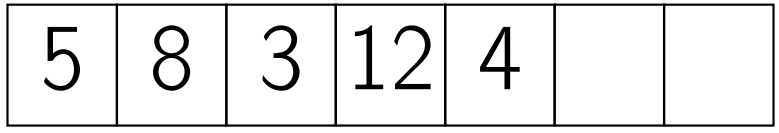

Times for Common Operations

	Add	Remove
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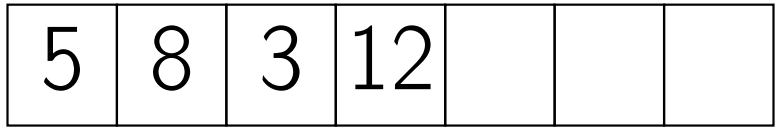
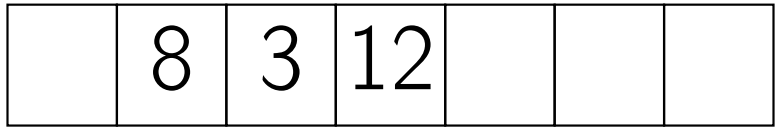
End	$O(1)$	$O(1)$
Middle		



Times for Common Operations

	Add	Remove
Beginning		$O(n)$
End	$O(1)$	$O(1)$
Middle		



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	Add	Remove
Beginning		$O(n)$
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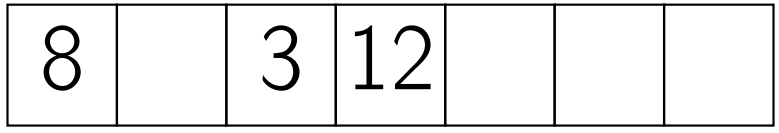
Middle		



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	Add	Remove
Beginning		$O(n)$
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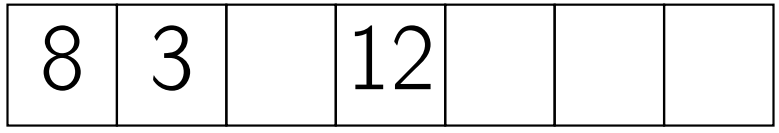
Middle		



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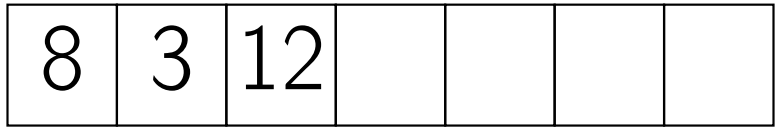
Middle		



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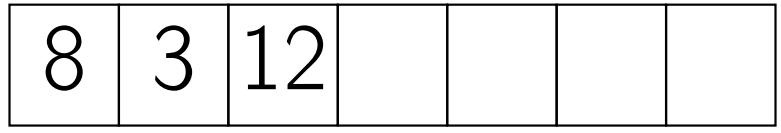

Times for Common Operations

	Add	Remove
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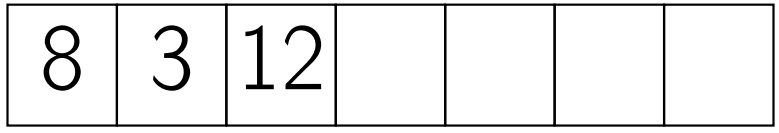
End	$O(1)$	$O(1)$
Middle	$O(n)$	$O(n)$



Summary

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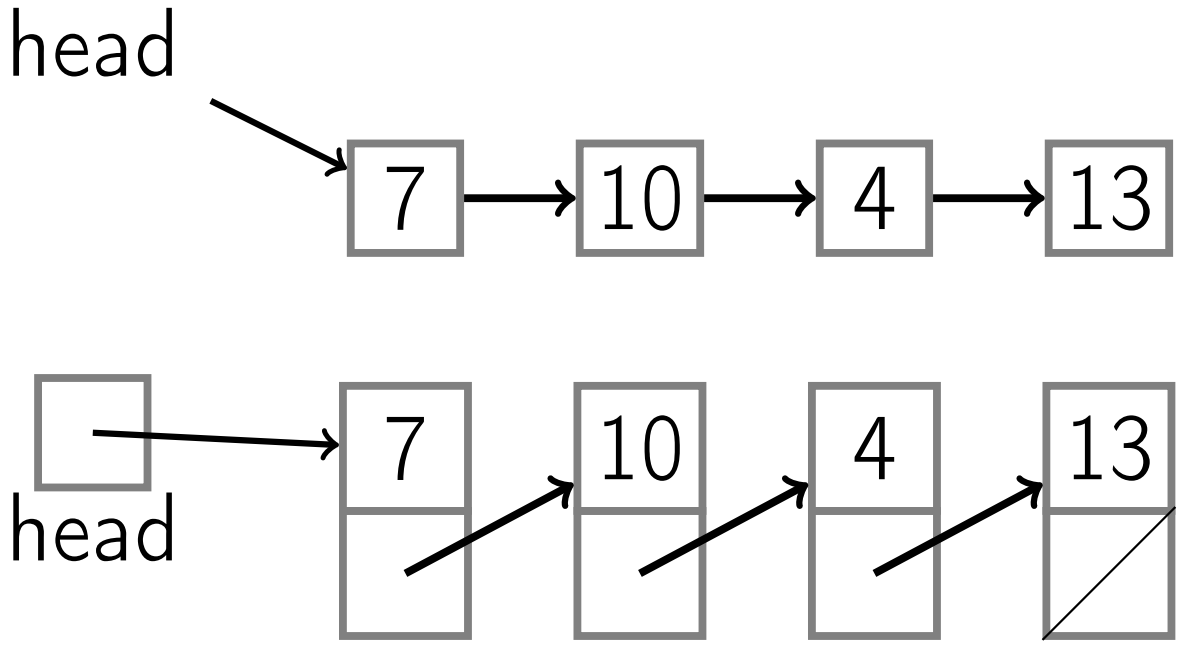
- Constant-time access to any element.
- Constant time to add/remove at the end.
- Linear time to add/remove at an arbitrary location.

Outline

Singly-Linked List



Node contains:

- `key`
- `next` pointer

List API

`PushFront(Key)` add to front

List API

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`Key TopFront()`	return front item

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`PushBack(Key)`	add to back
	also known as `Append`

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`PopFront()`	remove front item
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`AddBefore(Node, Key)`	adds key before node

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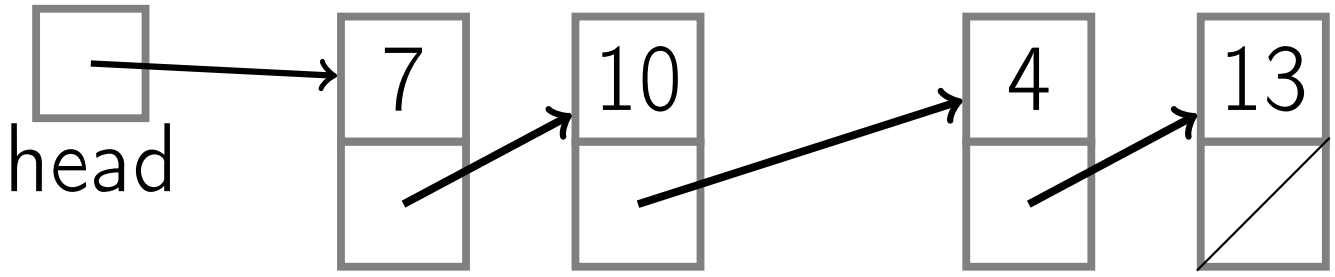
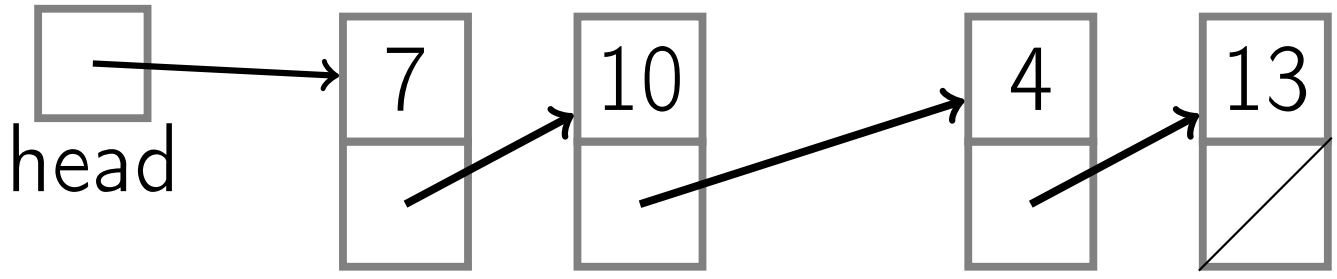
`AddBefore(Node, Key)`	adds key before node
`AddAfter(Node, Key)`	adds key after node

Times for Some Operations



Times for Some Operations

`PushFront`



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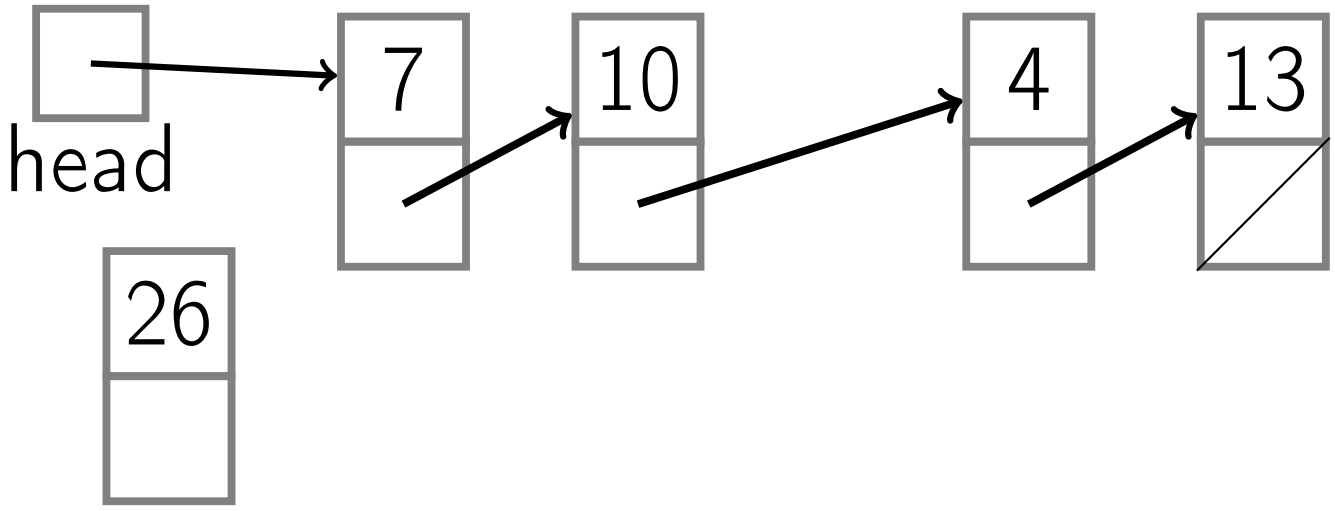

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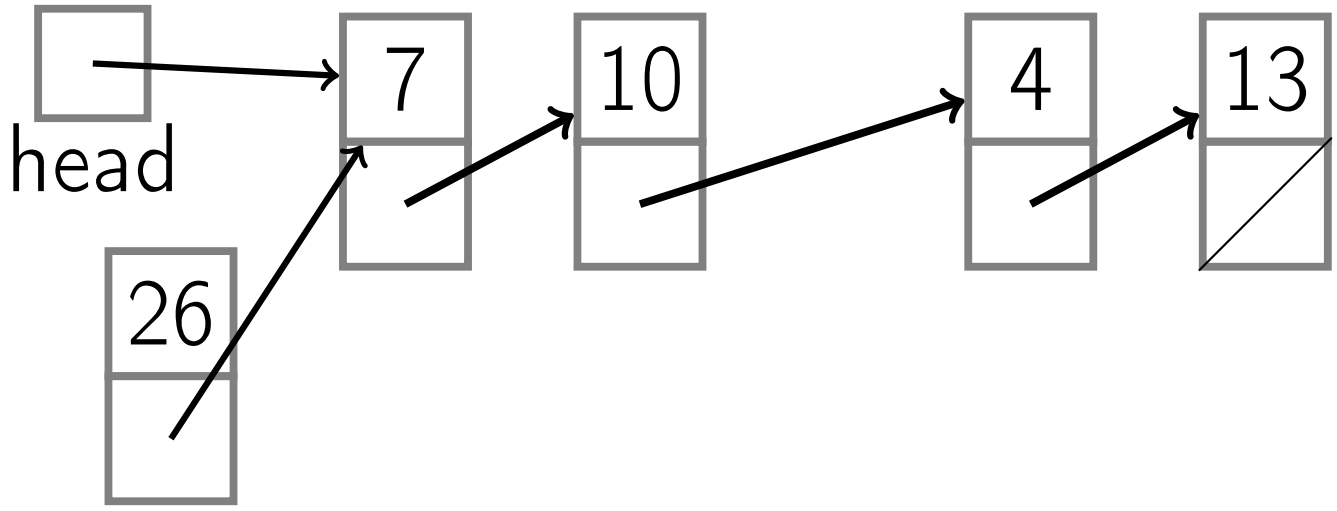
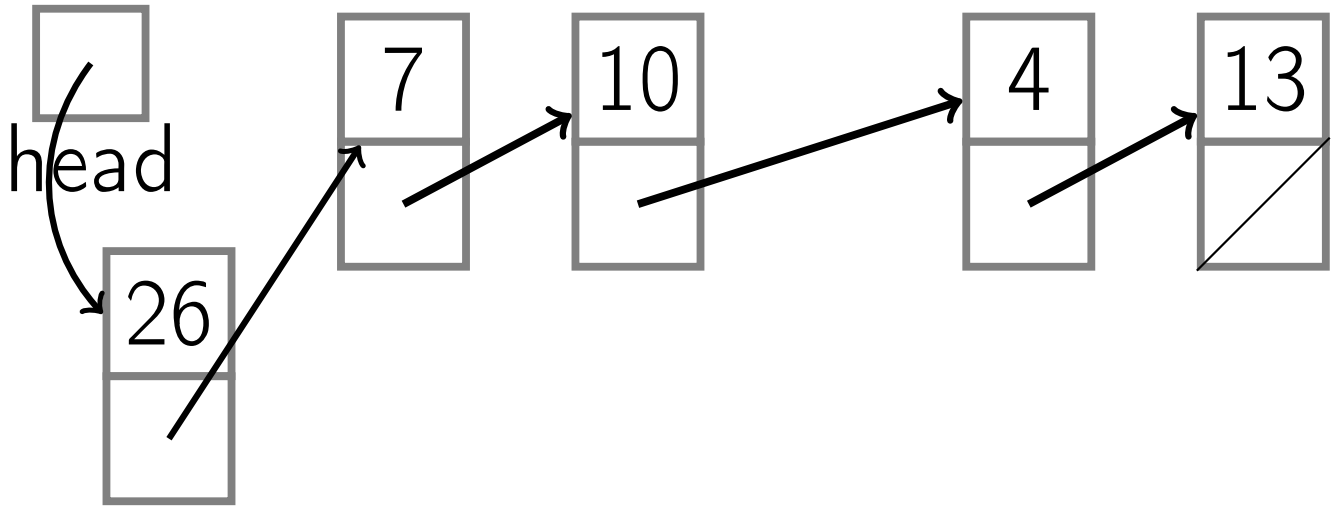


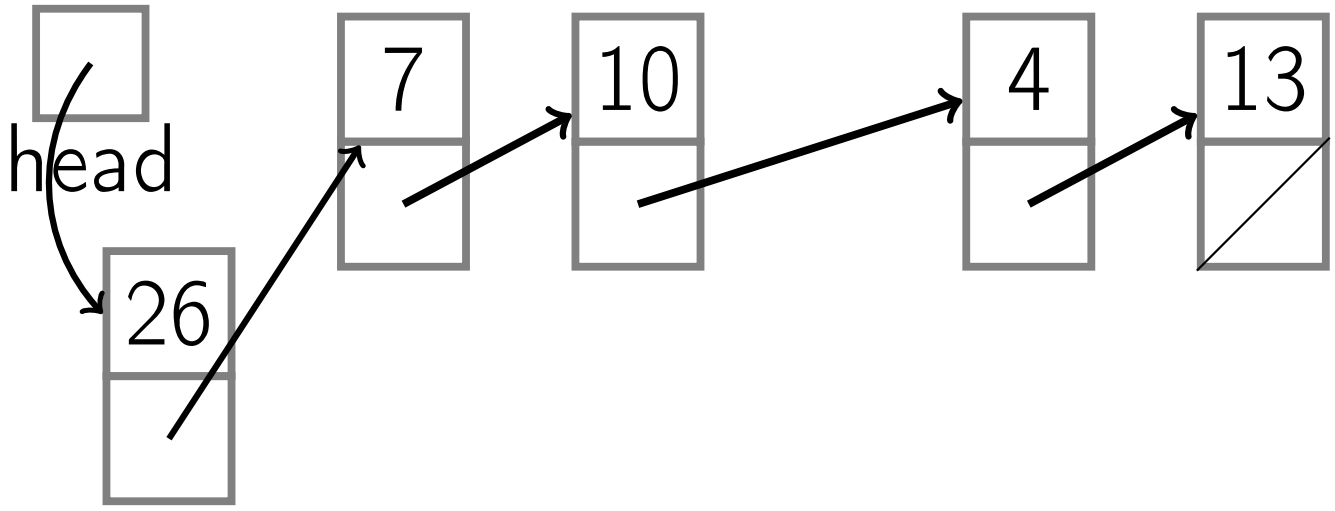
Times for Some Operations

`PushFront` $O(1)$



Times for Some Operations

`PopFront`



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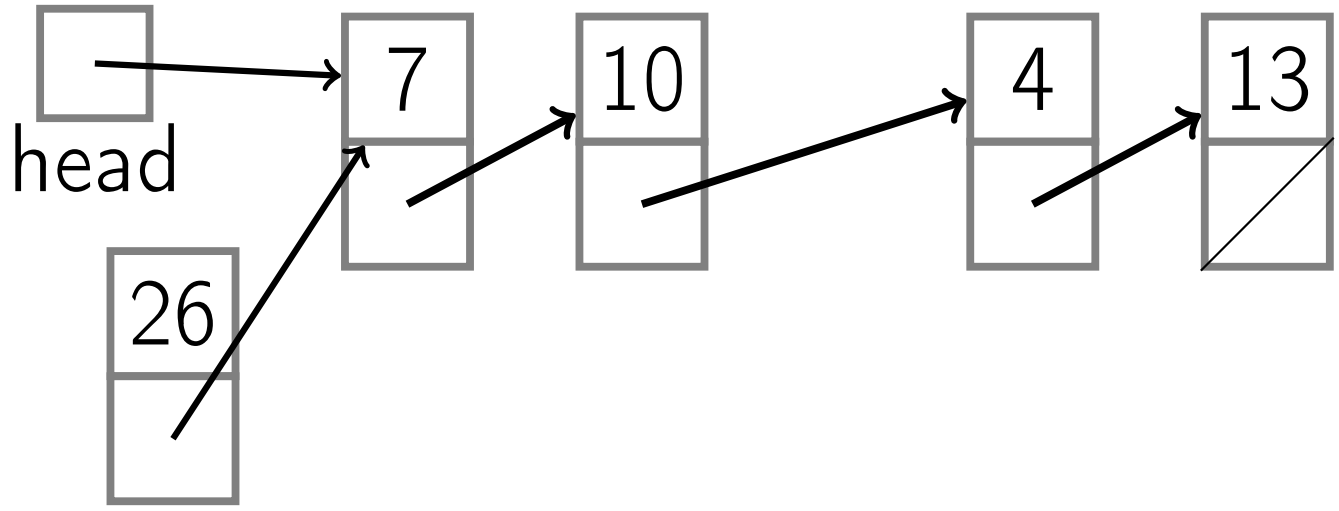

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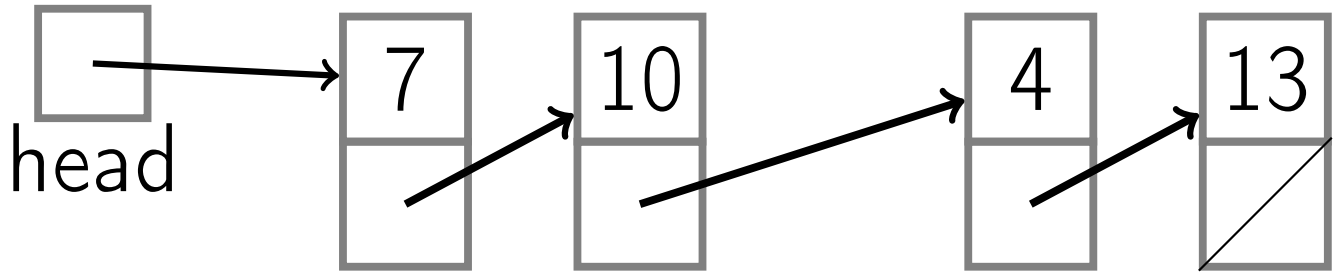
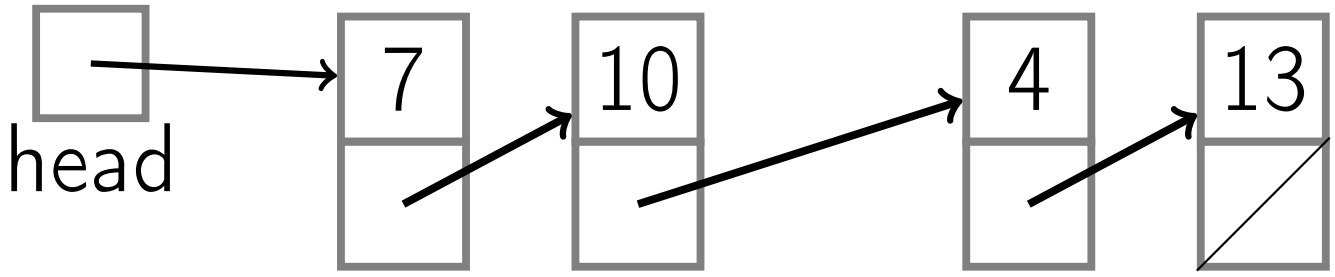


Times for Some Operations

`PushBack`

(no tail)



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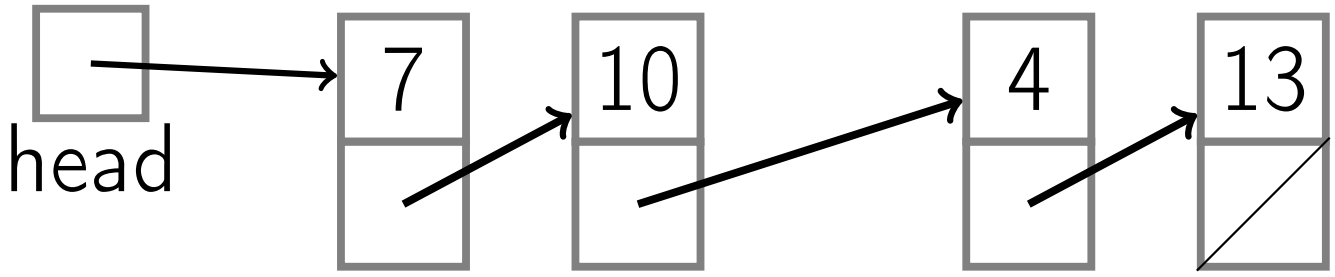

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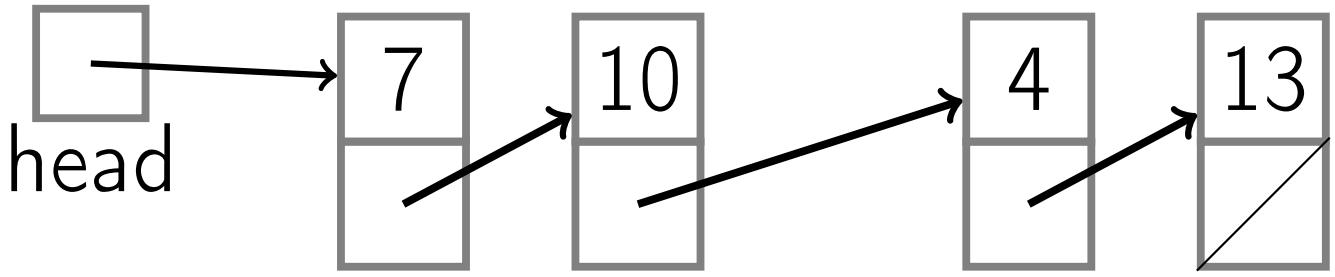

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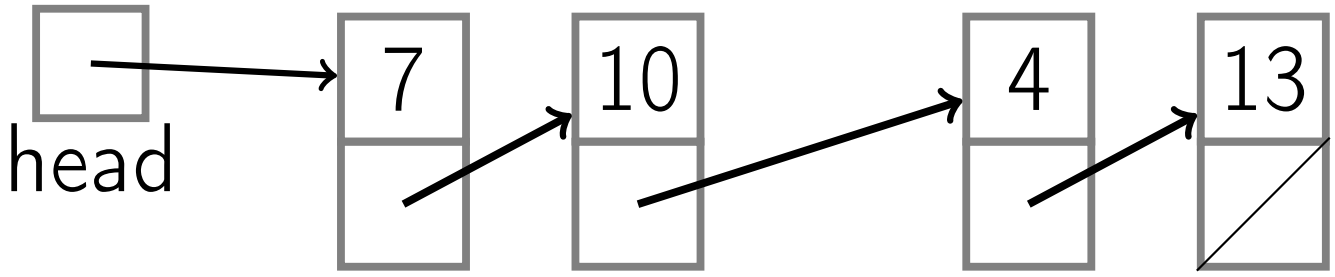

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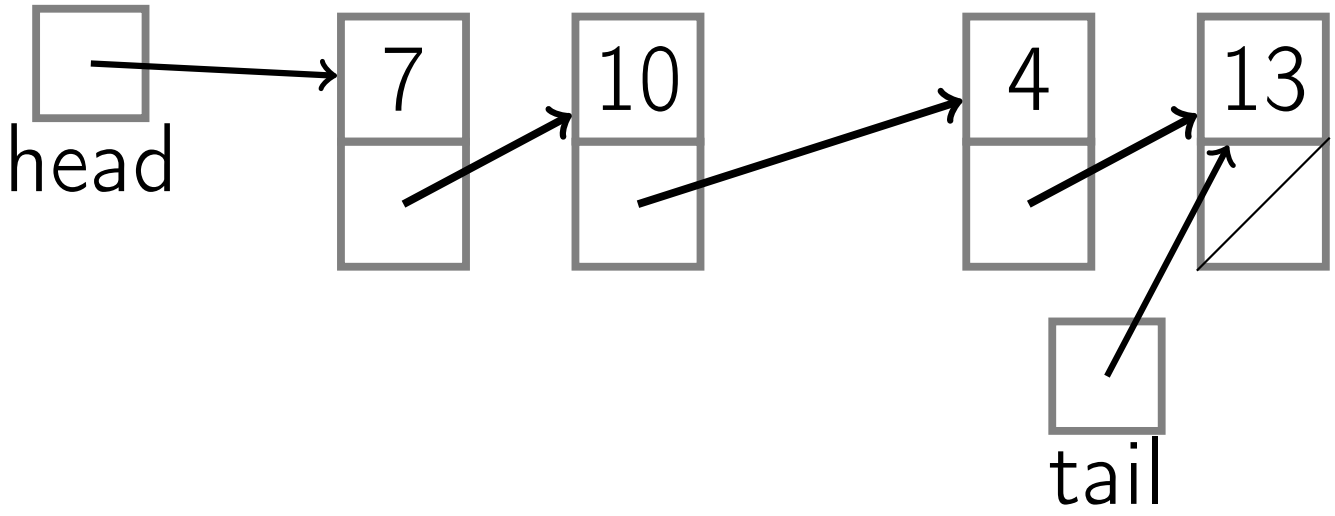
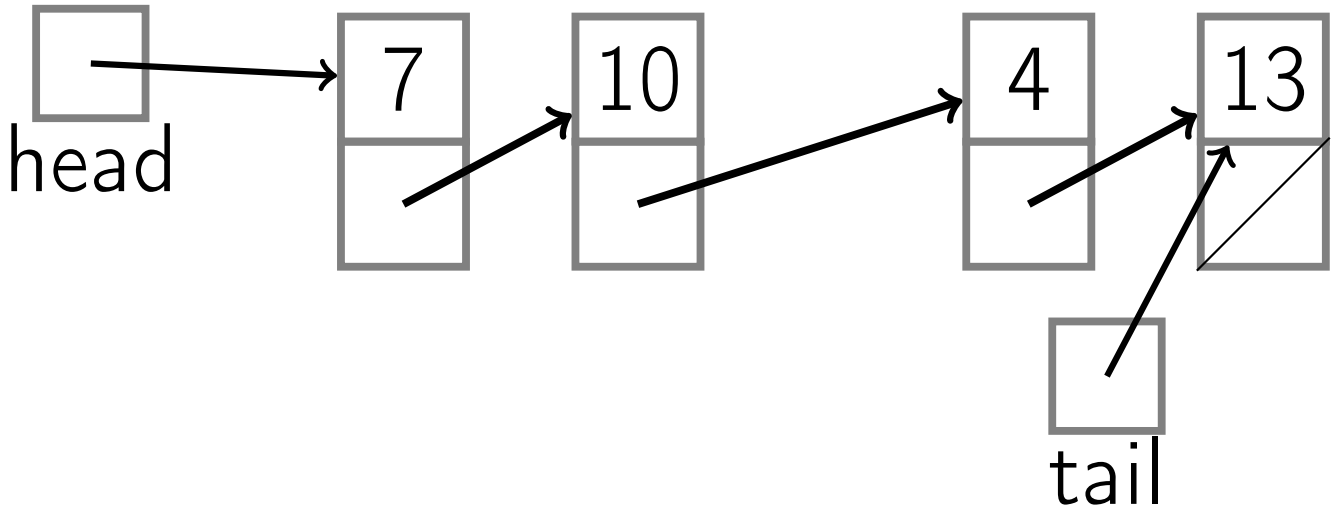


Times for Some Operations

`PushBack`
(with tail)



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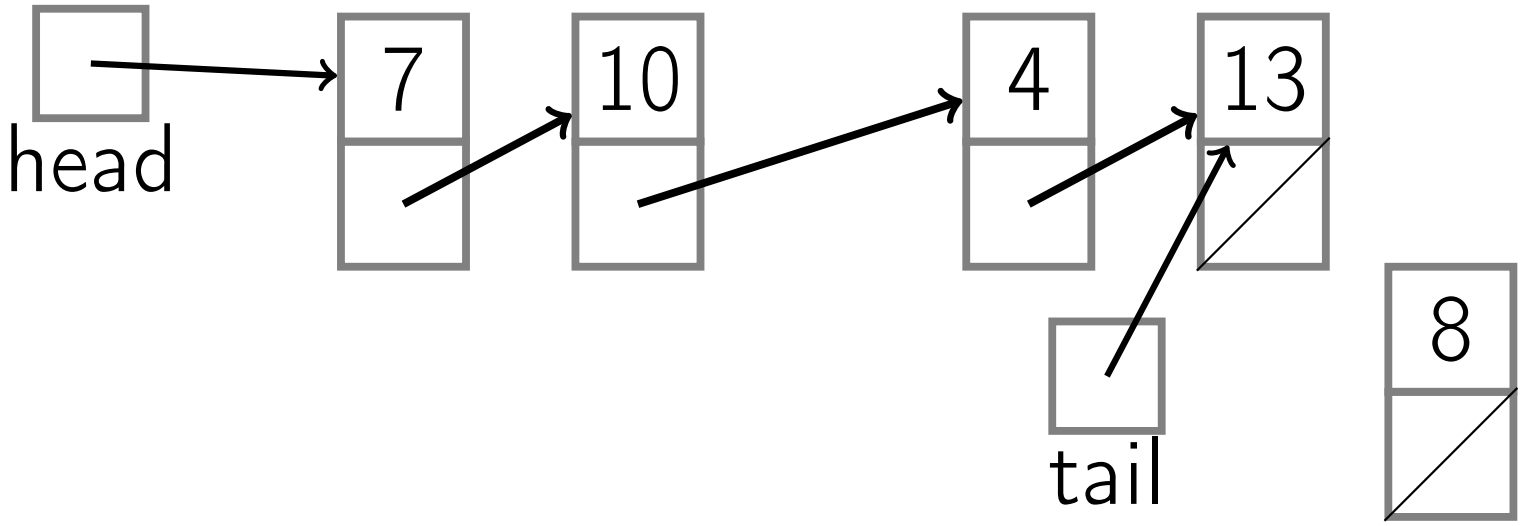

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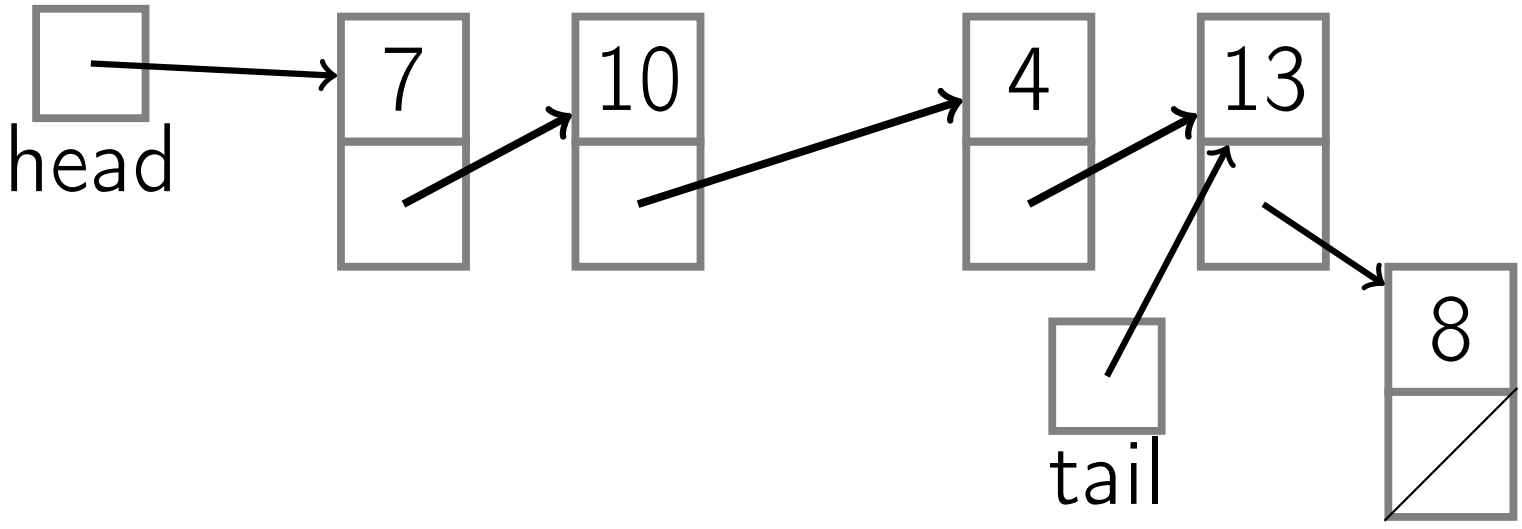

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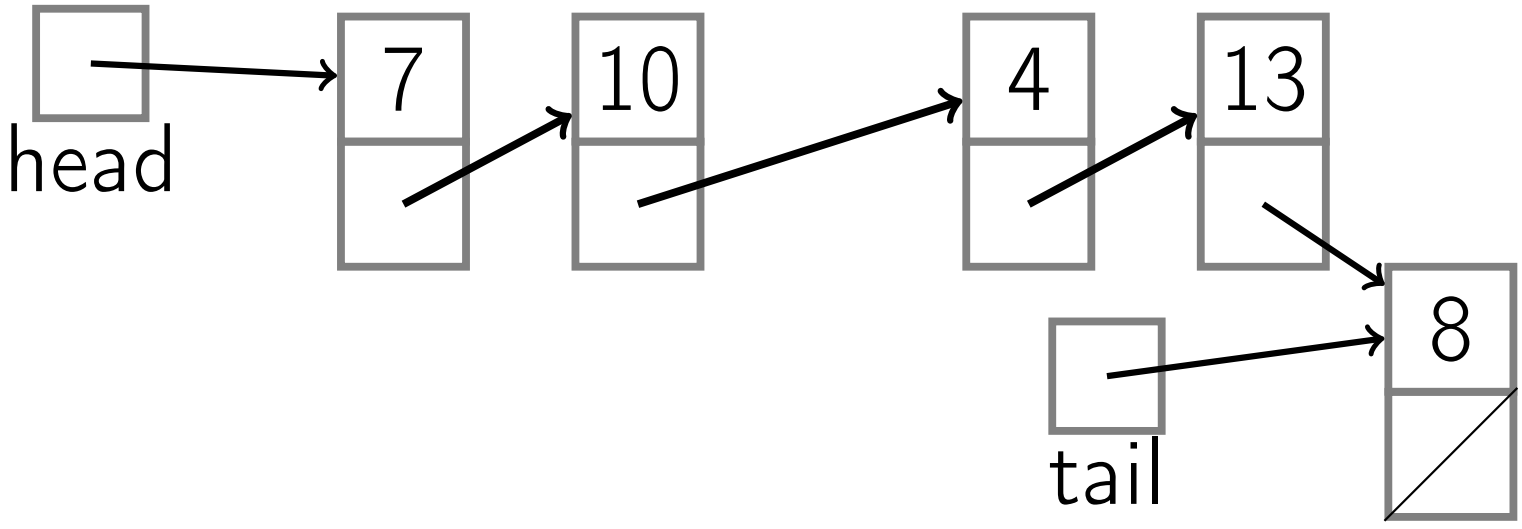
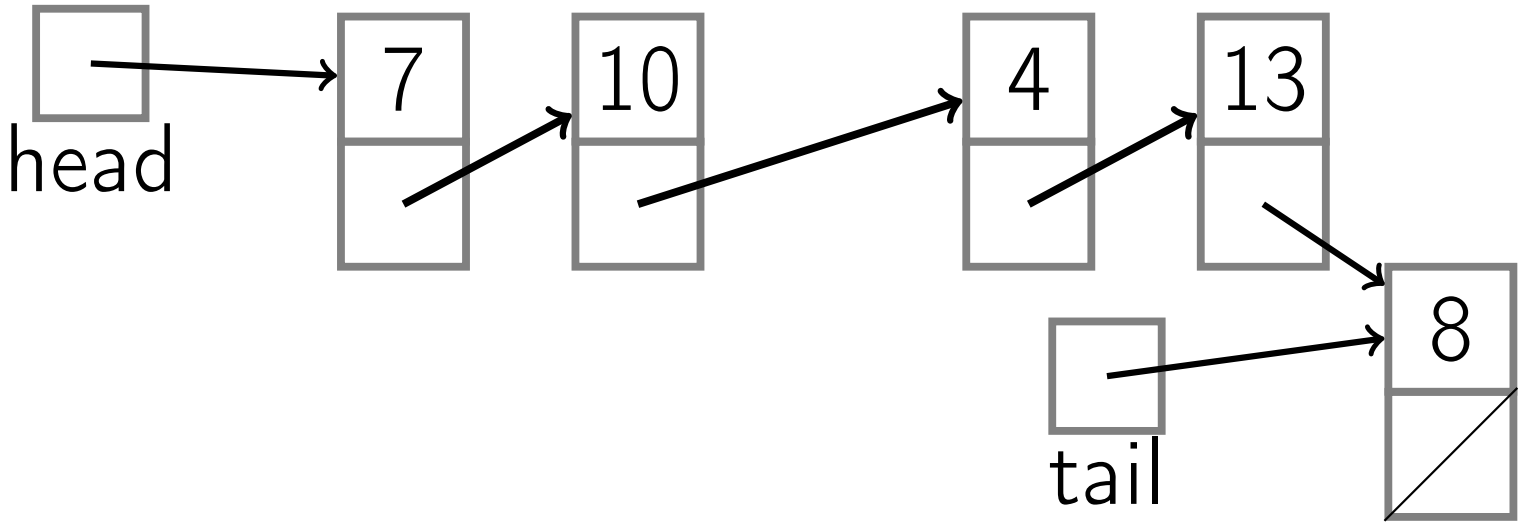
`PushBack` $O(1)$
(with tail)



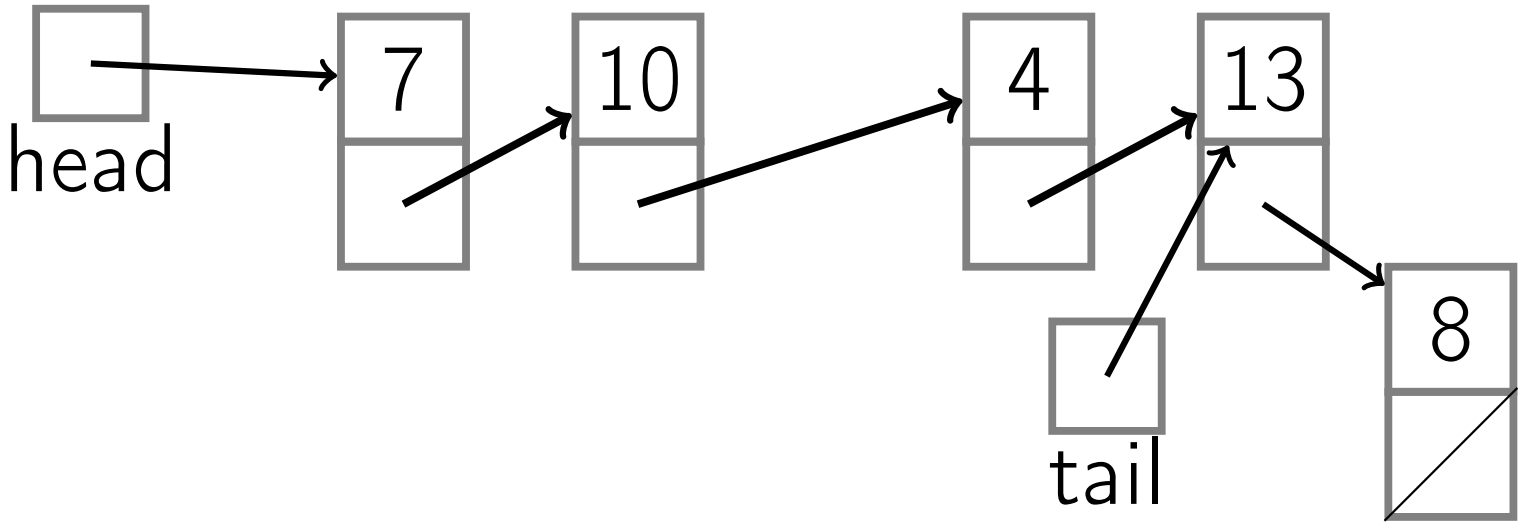
Times for Some Operations

PopBack
(with tail)



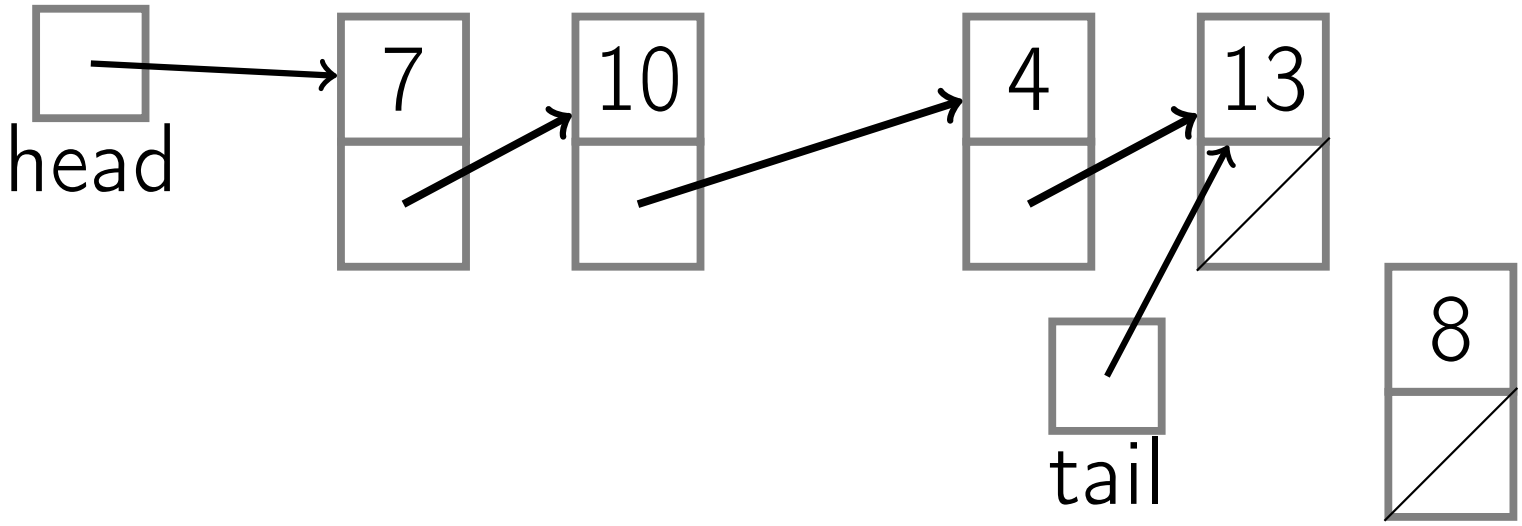
Times for Some Operations

PopBack
(with tail)



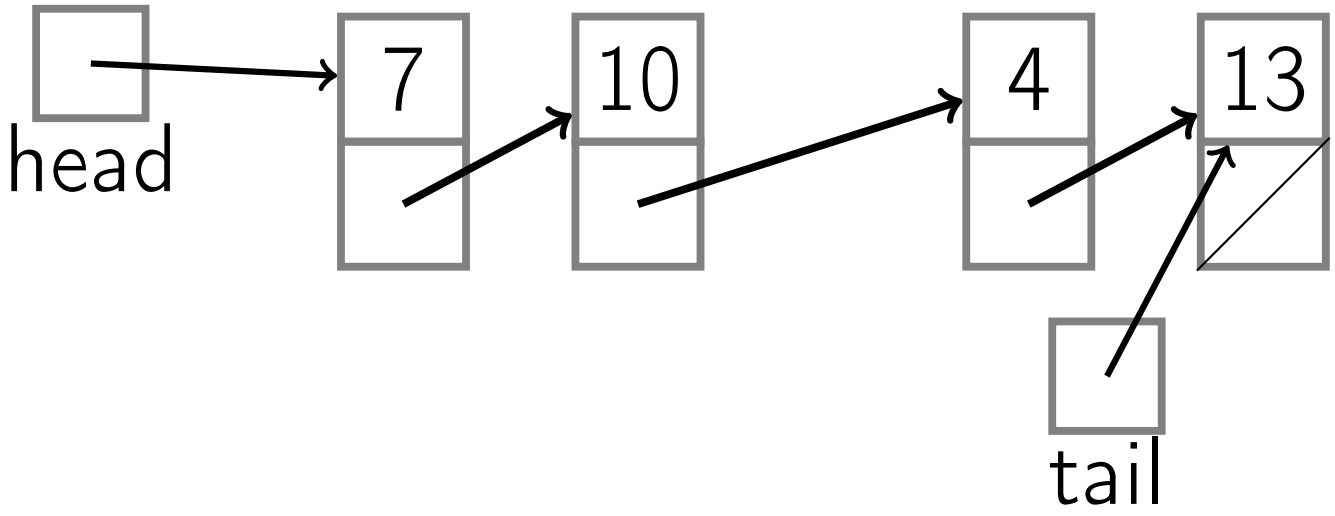
Times for Some Operations

PopBack
(with tail)



Times for Some Operations

`PopBack` $O(n)$
(with tail)



Singly-linked List

PushFront(*key*)

```
node ←new node
node.key ← key
node.next ← head
head ← node
if tail = nil:
    tail ← head
```

Singly-linked List

PopFront()

```
if head = nil:
  ERROR: empty list
head ← head.next
if head = nil:
  tail ← nil
```

Singly-linked List

PushBack(*key*)

node ←new node

node.key ← *key*

node.next =nil

Singly-linked List

PushBack(*key*)

```
node ←new node
node.key ← key
node.next =nil
if tail = nil:
    head ← tail ← node
```

Singly-linked List

PushBack(*key*)

```
node ←new node
node.key ← key
node.next =nil
if tail = nil:
    head ← tail ← node
else:
    tail.next ← node
    tail ← node
```

Singly-linked List

`PopBack()`

Singly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
```

Singly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
   head ← tail ←nil
```

Singly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
   head ← tail ←nil
else:
   p ← head
   while p.next.next ≠ nil:
      p ← p.next
```

Singly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
   head ← tail ←nil
else:
   p ← head
   while p.next.next ≠ nil:
      p ← p.next
   p.next ← nil; tail ← p
```

Singly-linked List

AddAfter(*node*, *key*)

```
node2 ←new node
node2.key ← key
node2.next = node.next
node.next = node2
if tail = node:
    tail ← node2
```

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	

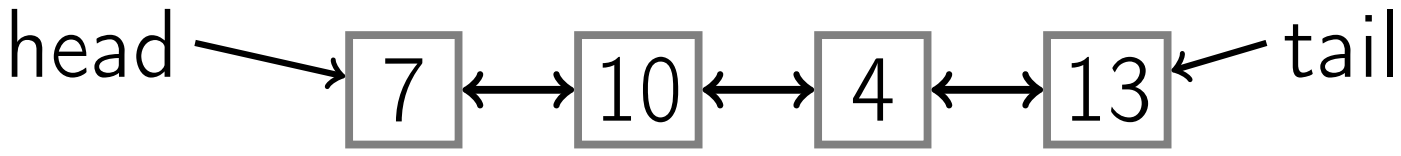
Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	

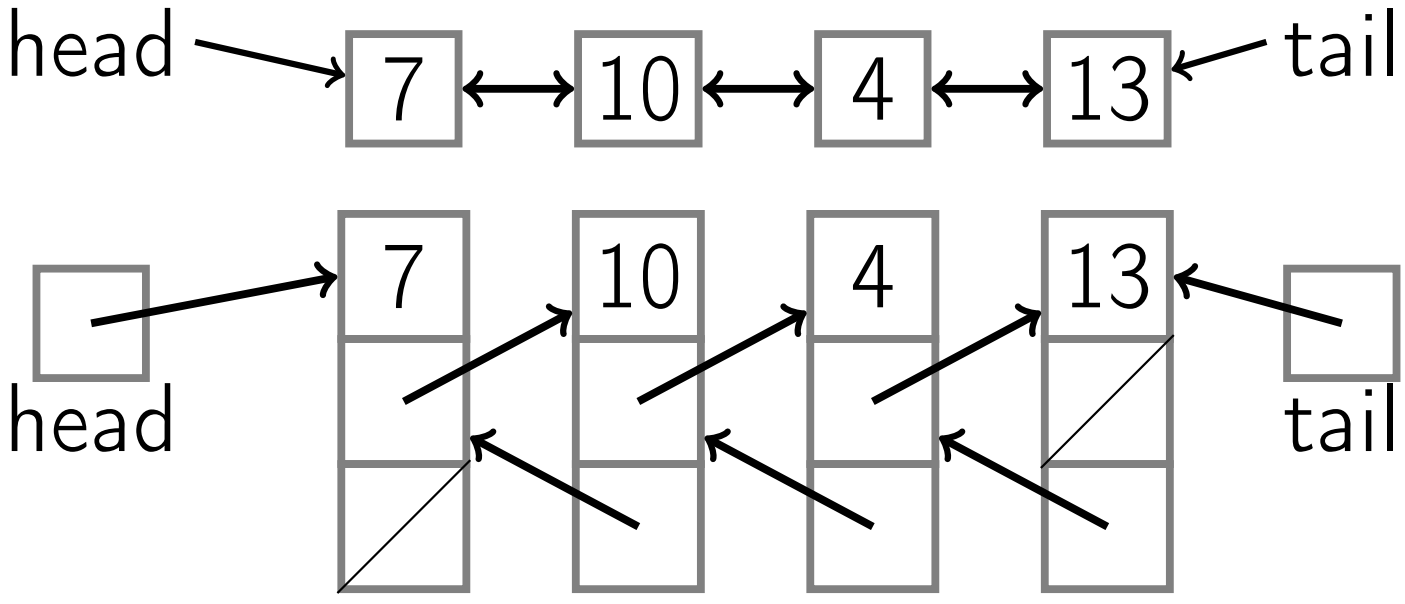
Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	
`AddAfter(Node, Key)`	$O(1)$	

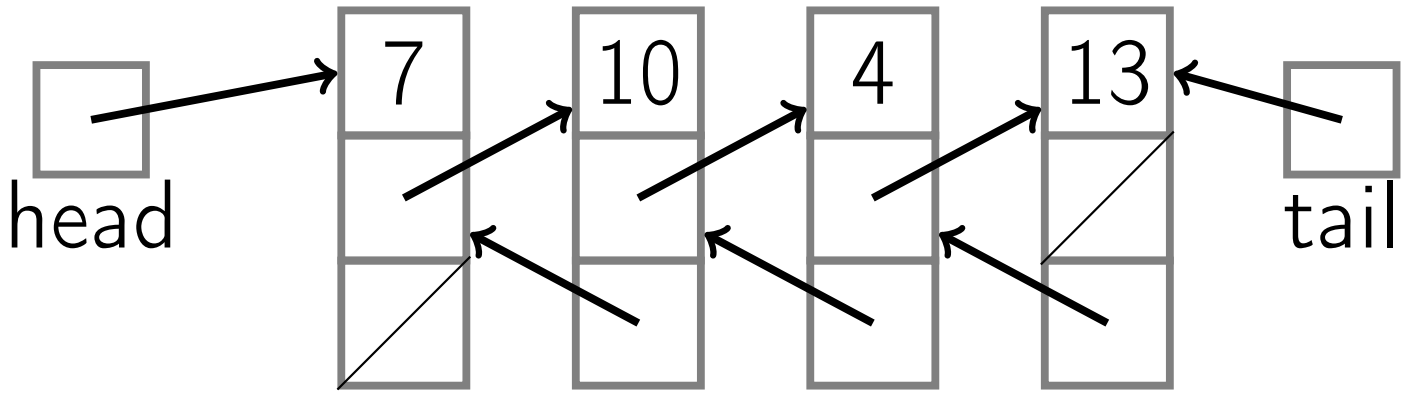
Doubly-Linked List



Doubly-Linked List



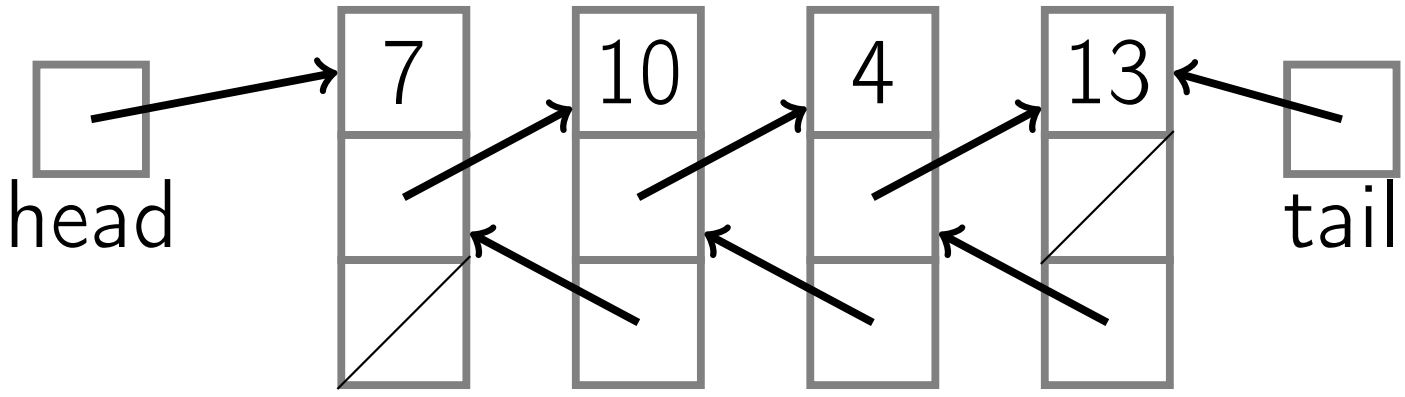
Doubly-Linked List



Node contains:

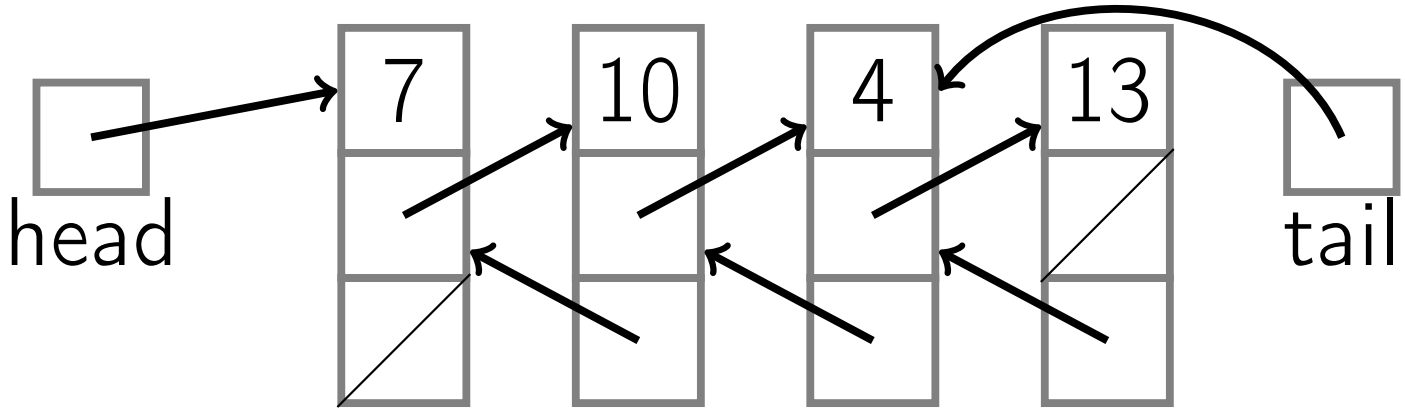
- `key`
- `next` pointer
- `prev` pointer

Doubly-Linked List



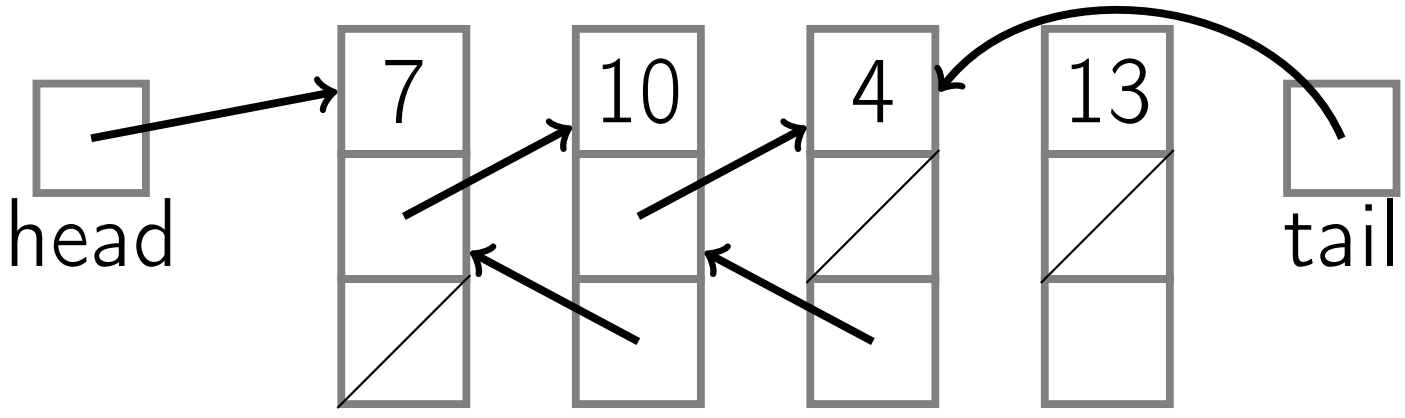
`PopBack`

Doubly-Linked List



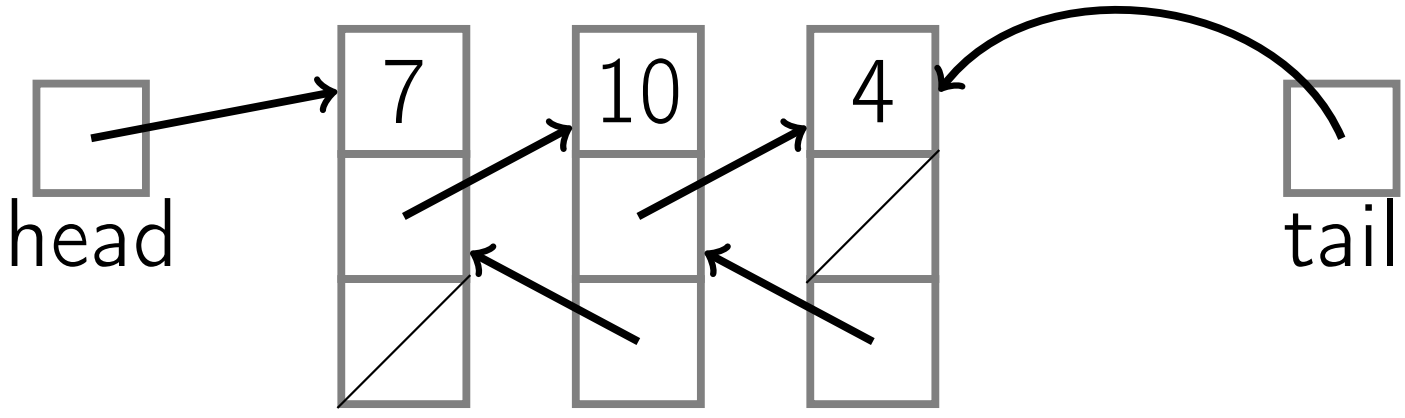
`PopBack`

Doubly-Linked List



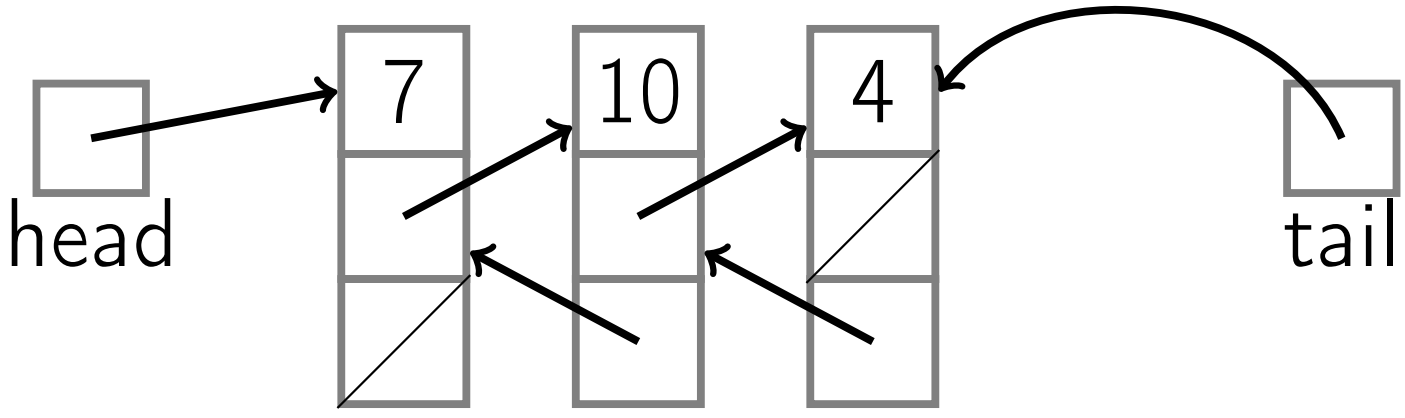
`PopBack`

Doubly-Linked List



`PopBack`

Doubly-Linked List



`PopBack` $O(1)$

Doubly-linked List

PushBack(*key*)

node $\leftarrow$`new node`

node.*key* $\leftarrow$ *key*; *node*.*next* $=$`nil`

Doubly-linked List

PushBack(*key*)

```
node ←new node
node.key ← key ;  node.next =nil
if  tail = nil:
    head ← tail ← node
    node.prev ←nil
```

Doubly-linked List

PushBack(*key*)

```
node ←new node
node.key ← key ;  node.next =nil
if  tail = nil:
    head ← tail ← node
    node.prev ←nil
else:
    tail.next ← node
    node.prev ← tail
    tail ← node
```

Doubly-linked List

`PopBack()`

Doubly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
```

Doubly-linked List

PopBack()

```
if head = nil:   ERROR: empty list
if head = tail:
   head ← tail ←nil
```

Doubly-linked List

PopBack()

```
if head = nil:  ERROR: empty list
if head = tail:
   head ← tail ←nil
else:
   tail ← tail.prev
   tail.next ←nil
```

Doubly-linked List

AddAfter(*node*, *key*)

```
node2 ←new node
node2.key ← key
node2.next ← node.next
node2.prev ← node
node.next ← node2
if node2.next ≠nil:
    node2.next.prev ← node2
if tail = node:
    tail ← node2
```

Doubly-linked List

AddBefore(*node*, *key*)

```
node2 ←new node
node2.key ← key
node2.next ← node
node2.prev ← node.prev
node.prev ← node2
if node2.prev ≠nil:
    node2.prev.next ← node2
if head = node:
    head ← node2
```

Singly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	$O(n)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	$O(n)$	
`AddAfter(Node, Key)`	$O(1)$	

Doubly-Linked List	no tail	with tail
`PushFront(Key)`	$O(1)$	
`TopFront()`	$O(1)$	
`PopFront()`	$O(1)$	
`PushBack(Key)`	$O(n)$	$O(1)$
`TopBack()`	$O(n)$	$O(1)$
`PopBack()`	~~$O(n)$~~ $O(1)$	
`Find(Key)`	$O(n)$	
`Erase(Key)`	$O(n)$	
`Empty()`	$O(1)$	
`AddBefore(Node, Key)`	~~$O(n)$~~ $O(1)$	
`AddAfter(Node, Key)`	$O(1)$	

Summary

- Constant time to insert at or remove from the front.

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- With tail and doubly-linked, constant time to insert at or remove from the back.

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- $O(n)$ time to find arbitrary element.
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Summary

- Constant time to insert at or remove from the front.
- With tail and doubly-linked, constant time to insert at or remove from the back.
- $O(n)$ time to find arbitrary element.
- List elements need not be contiguous.
- With doubly-linked list, constant time to insert between nodes or remove a node.