Binary Search Trees: Basic Operations

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Implement basic operations on Binary Search Trees.
- Understand some of the difficulties with making updates.

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Find

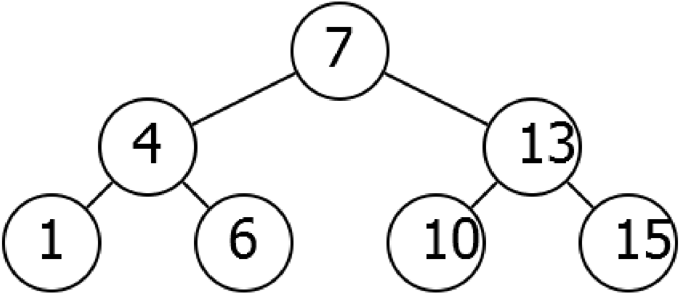
Find

Input: Key k, Root R

Output: The node in the tree of R with key k

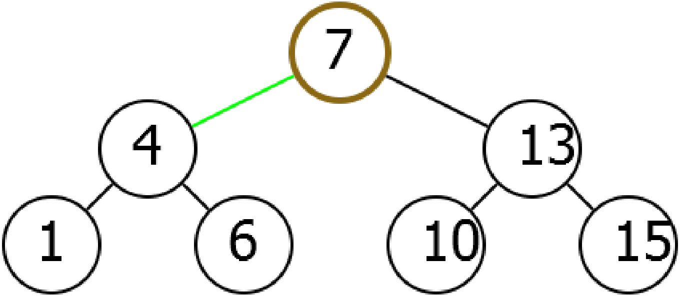
Idea

`Find(6)`



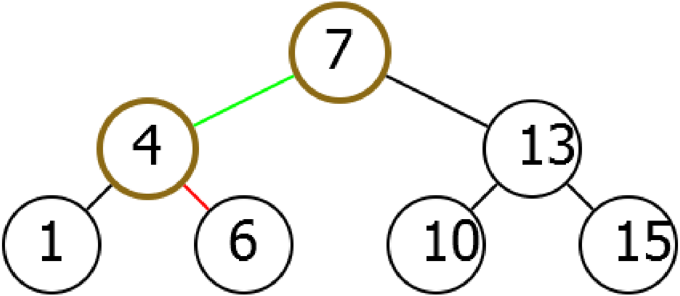
Idea

`Find(6)`



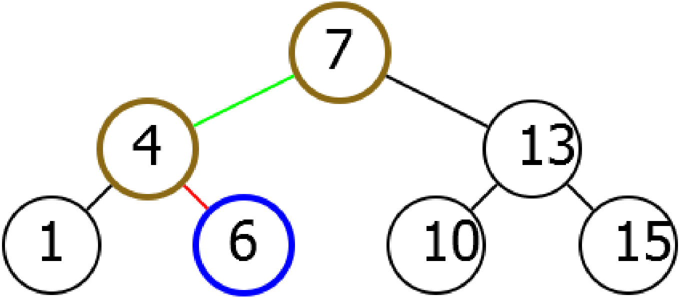
Idea

`Find(6)`



Idea

`Find(6)`



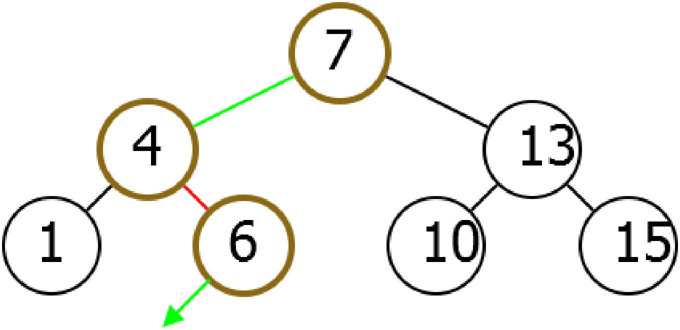
Algorithm

$\texttt{Find}(k, R)$

```
if R.Key = k:
  return R
else if R.Key > k:
  return Find(k, R.Left)
else if R.Key < k:
  return Find(k, R.Right)
```

Missing Key

Run `Find`(5).



Key not in tree. Did find point where it should be.

Missing Key

If you stop before reaching a null pointer, you find the place in the tree where k would fit.

Modification

Find (modified)

```
else if R.Key > k :
  if R.Left ≠ null:
    return Find(k, R.Left)
  return R
```

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Adjacent Elements

Given a node N in a Binary Search Tree, would like to find adjacent elements.

Next

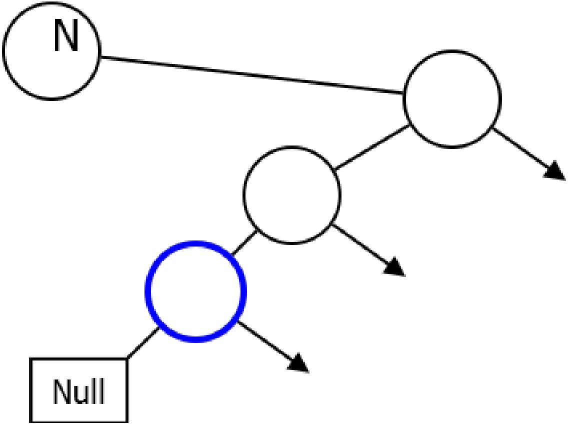
Next

Input: Node N

Output: The node in the tree with the next largest key.

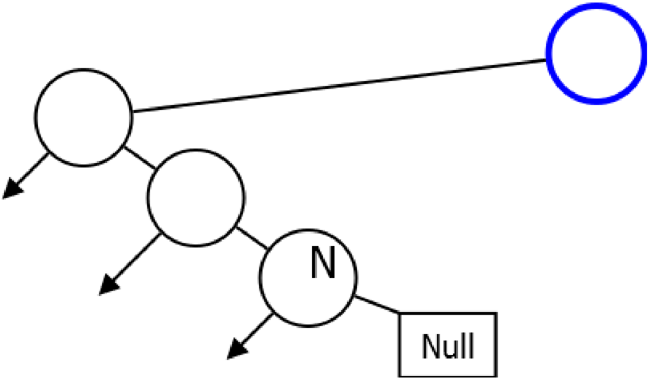
Case I

If you have right child.



Case II

No right child.



Next

```
Next(N)

if N.Right ≠ null:
  return LeftDescendant(N.Right)
else:
  return RightAncestor(N)
```

Left Descendant

LeftDescendant(N)

```
if N.Left = null
  return N
else:
  return LeftDescendant(N.Left)
```

Right Ancestor

RightAncestor(N)

```
if N.Key < N.Parent.Key
  return N.Parent
else:
  return RightAncestor(N.Parent)
```

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Range Search

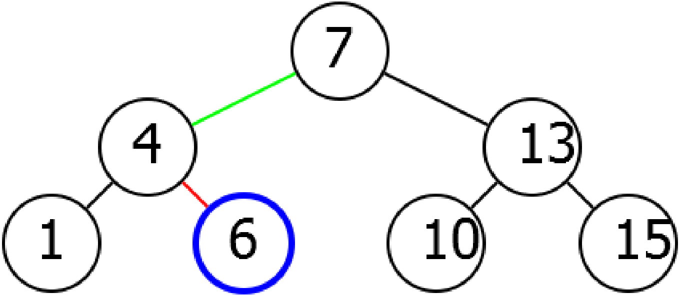
Range Search

Input: Numbers x, y, root R

Output: A list of nodes with key between x and y

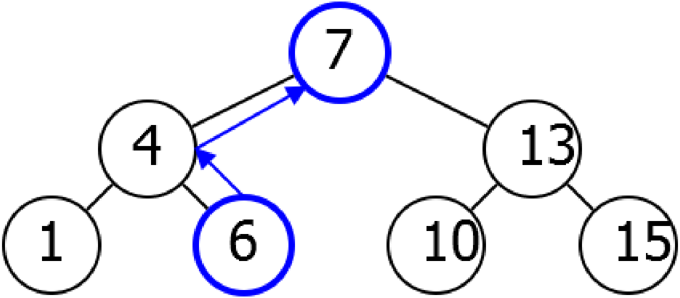
Idea

RangeSearch$(5, 12)$.



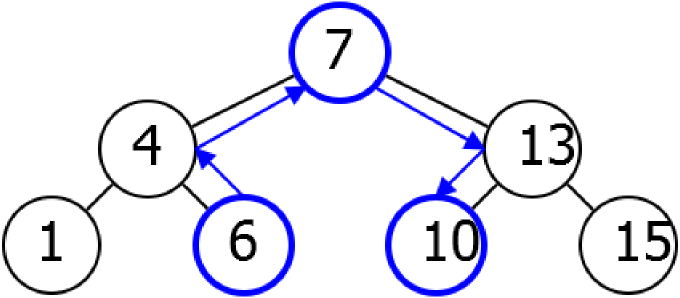
Idea

$\texttt{RangeSearch}(5, 12)$.



Idea

`RangeSearch`$(5, 12)$.



Implementation

RangeSearch(x, y, R)

```
L ← ∅
N ← Find(x, R)
while N.Key ≤ y
   if N.Key ≥ x:
      L ← L.Append(N)
   N ← Next(N)
return L
```

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Insert

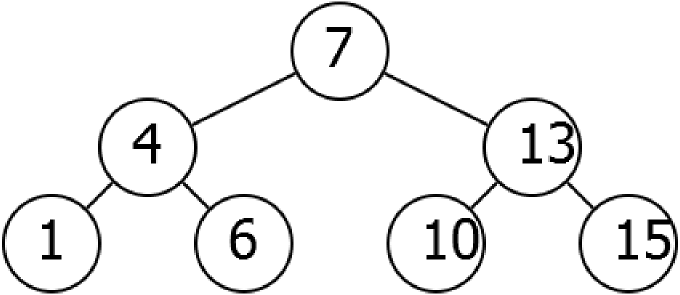
Insert

Input: Key k and root R

Output: Adds node with key k to the tree

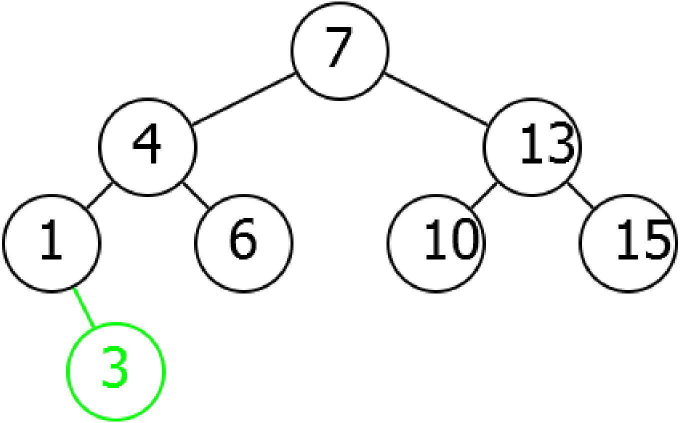
Insert Idea

`Insert(3)`



Insert Idea

`Insert(3)`



Implementation

Insert(k, R)

$P \leftarrow$ Find(k, R)

Add new node with key k as child of P

Outline

1. Find
2. Next Element
3. Search
4. Insert
5. Delete

Delete

Delete

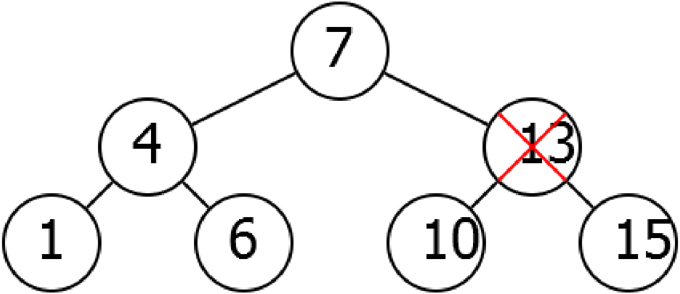
Input: Node N

Output: Removes node N from the tree

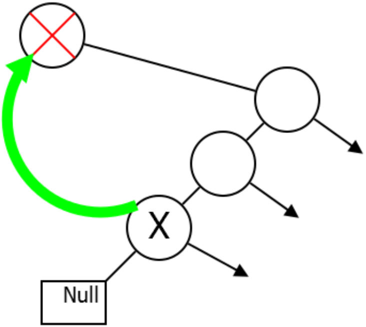
Difficulty

Cannot simply remove.

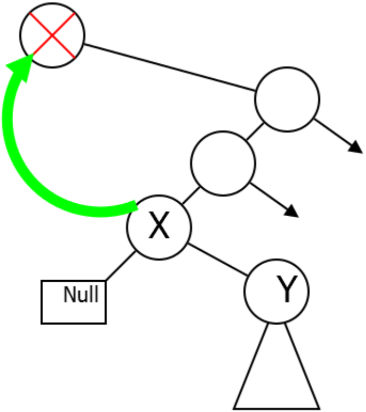
`Delete(13)`



Idea



Idea



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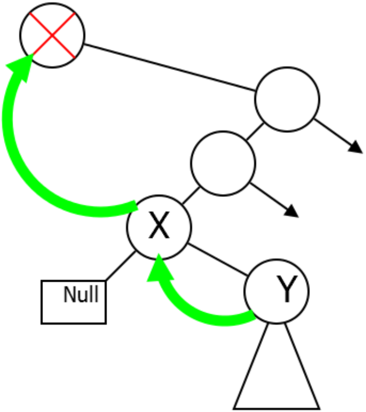
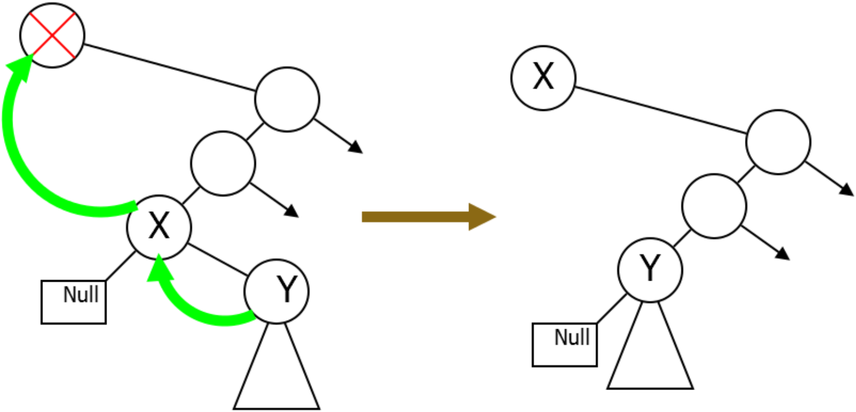


Idea



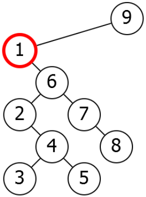
Implementation

Delete(N)

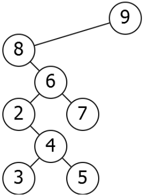
```
if N.Right = null:
    Remove N, promote N.Left
else:
    X ← Next(N)
\\  X.Left = null
    Replace N by X, promote X.Right
```

Problem

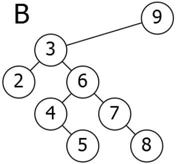
Which of the following trees is obtained when the selected node is deleted?



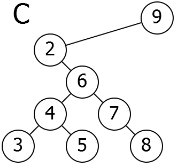
A



B



C



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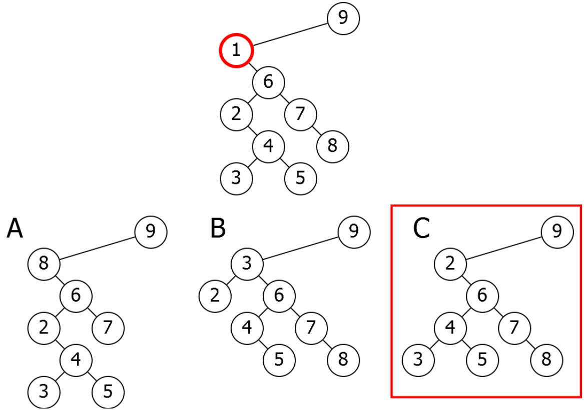

Next Time

Runtime and balance.