Binary Search Trees: Balance

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Data Structures
Data Structures and Algorithms

Learning Objectives

- Think about the runtime of basic binary tree operations.
- Understand the motivation behind binary search tree balance.
- Implement a rotation.

Outline

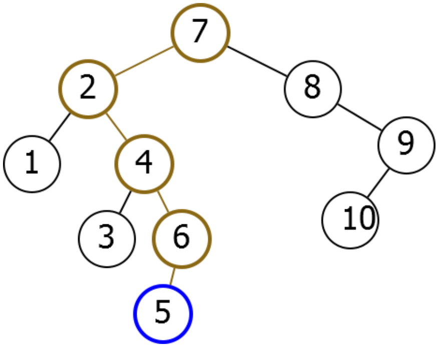
1. Runtime
2. Balanced Trees
3. Rotations

Runtime

How long do Binary Search Tree operations take?

Find

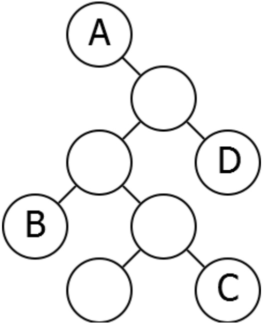
`Find(5)`



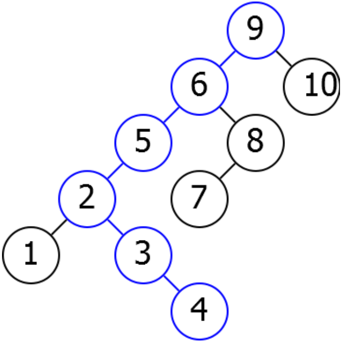
Number of operations = $O(\text{Depth})$

Problem

Which nodes will be faster to search for in the following tree?



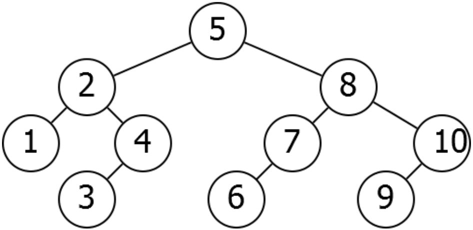
Example I



Depth can be as bad as n.

Outline

Example II



Depth can be much smaller.

Balance

- Want left and right subtrees to have approximately the same size.

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Balance

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- Suppose perfectly balanced:
  - Each subtree half the size of its parent.
  - After $\log_2(n)$ levels, subtree of size 1.
  - Operations run in $O(\log(n))$ time.

Problem

Insertions and deletions can destroy balance!

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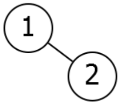

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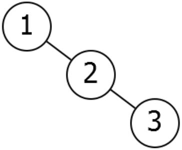

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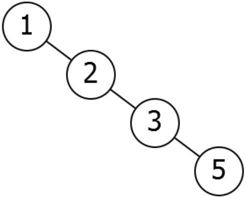

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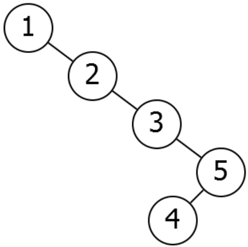

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Rebalancing

Idea: Rearrange tree to maintain balance.

Rebalancing

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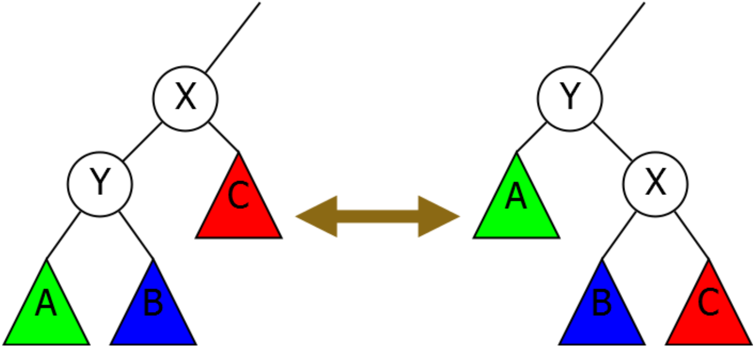
Problem: How do we rearrange tree while maintaining order?

Rotations



$$A < Y < B < X < C$$

Implementation

RotateRight(X)

$P \leftarrow X.\texttt{Parent}$
$Y \leftarrow X.\texttt{Left}$
$B \leftarrow Y.\texttt{Right}$
$Y.\texttt{Parent} \leftarrow P$
$P.\texttt{AppropriateChild} \leftarrow Y$
$X.\texttt{Parent} \leftarrow Y, Y.\texttt{Right} \leftarrow X$
$B.\texttt{Parent} \leftarrow X, X.\texttt{Left} \leftarrow B$

Next Time

How to keep a tree balanced. AVL trees.